Preliminary Edition:

Matrices, Geometry&*Mathematica*

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MGM.02 2D Matrix Action
Literacy Sheet

What you need to know when you're away from the machine.

L.1)

Here is a random 2D matrix:

```
A = {{N[Random[Real, {-2, 2}], 3], N[Random[Real, {-2, 2}], 3]},
  {N[Random[Real, {-2, 2}], 3], N[Random[Real, {-2, 2}], 3]}};

MatrixForm[A]
```

$\begin{pmatrix} 1.93 & -1.44 \\ -0.0975 & 0.586 \end{pmatrix}$

The first horizontal row of A is:

{..............., ...}.

The second horizontal row of A is:

{..............., ...}.

L.2)

Here is a random 2D matrix:

```
A = {{N[Random[Real, {-2, 2}], 3], N[Random[Real, {-2, 2}], 3]},
  {N[Random[Real, {-2, 2}], 3], N[Random[Real, {-2, 2}], 3]}};
MatrixForm[A]
```

$\begin{pmatrix} -0.867 & -0.251 \\ -1.45 & -0.169 \end{pmatrix}$

The first vertical column of A is:

{..............., ...}.

The second vertical column of A is:

{..............., ...}.

L.3)

Here is a 2D matrix A:

$A = \begin{pmatrix} 2.0 & 1.0 \\ -1.0 & 0.0 \end{pmatrix}$.

And here's a random 2D vector X:

```
X = {N[Random[Real, {-2, 2}], 1], N[Random[Real, {-2, 2}], 1]}
```

{-1., -1.}

When you hit X with A by calculating A.X, you get

A . X = {............, ...}.

L.4)

Here is a 2D matrix A:

$A = \begin{pmatrix} 2.7 & 0.5 \\ -1.4 & 1.3 \end{pmatrix}$.

And here's a random 2D vector X:

```
X = {N[Random[Real, {-2, 2}], 1], N[Random[Real, {-2, 2}], 1]}
```

{-0.1, -0.5}

.There are numbers a and b so that you can calculate A . X via

$A . X = a\{2.7, -1.4\} + b\{0.5, 1.3\}$.

The correct numbers a and b are

a = ...

b =........................ .

L.5)

Here is a random 2D matrix:

```
A = {{N[Random[Real, {-2, 2}], 3], N[Random[Real, {-2, 2}], 3]},
  {N[Random[Real, {-2, 2}], 3], N[Random[Real, {-2, 2}], 3]}};
MatrixForm[A]
```

$\begin{pmatrix} -1.51 & -0.345 \\ 1.15 & -1.95 \end{pmatrix}$

The numbers a, b, c and d that make

$A^t = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

are

a = ... b = ...
c =....................... d = ...

L.6)

Given a 2D matrix A, then the horizontal rows of
A^t (= Transpose[A])
are the vertical columns of A.
Agree..........
Disagree.......

L.7)

Given a 2D matrix A, then the vertical columns of
A^t (= Transpose[A])
are the horizontal rows of A.
Agree..........
Disagree.......

L.8)

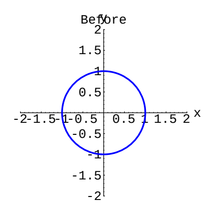
When you hit a circle with a matrix you get
a) Another circle.
b) An ellipse
c) An ellipse,a line or just one point
d) A hyperbola, ellipse,a line or just one point.
Your response:.............

L.9)

When you hit a rectangle with a matrix you get
a) Another rectangle.
b) An ellipse
c) An ellipse,a line or just one point
d) A parallelogram, a line or just one point.
Your response:.............

L.10)

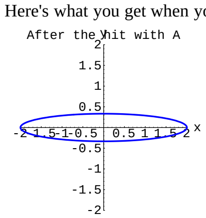
Here's the unit circle waiting to be hit with a matrix:



Here's an xystretcher A:

$\begin{pmatrix} 2 & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$

Here's what you get when you hit the unit circle A



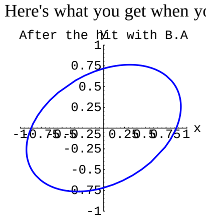
Given that the area of the unit circle measures out to π square units, what does the area of the ellipse plotted above measure out to?

L.11)

Go with this matrix A:

$\begin{pmatrix} 1 & 0 \\ 0 & \frac{2}{3} \end{pmatrix}$

B is a rotation matrix whose hits rotate everything by $\frac{\pi}{6}$ counterclockwise radians.
Here's what you get when you hit the unit circle with B.A:



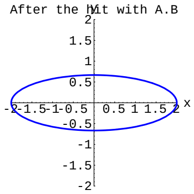
Given that the area of the unit circle measures out to π square units, what does the area of the ellipse plotted above measure out to?

□L.12)

Go with this matrix A:

$$\begin{pmatrix} 2 & 0 \\ 0 & \frac{2}{3} \end{pmatrix}$$

B is the rotation matrix whose hits rotate everything by $\frac{\pi}{4}$ counterclockwise radians.
Here's what you get when you hit the unit circle with A.B:



Given that the area of the unit circle measures out to π square units, what does the area of the ellipse plotted above measure out to?
Why is this ellipse not tilted?

□L.13)

For a given 2D matrix A, the inverse matrix A^{-1} is the matrix you hit with to undo whatever a hit with A did.
For instance, when you hit with this stretcher matrix A:

$$\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$$

The numbers a, b, c and d that make

$$A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

are

a = ... b = ...
c =........................ d = ...

□L.14)

Hits with the following matrix rotate everything about {0,0} by s counterclockwise radians:

$$\begin{pmatrix} \text{Cos}[s] & -1.\ \text{Sin}[s] \\ \text{Sin}[s] & \text{Cos}[s] \end{pmatrix}$$

The entries a, b, c and d that make

$$\text{rotator}[s]^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

are

a = ... b = ...
c =........................ d = ...

□L.15)

Remembering that

$$\text{shear}[a]\,.\,\text{shear}[b] = \text{shear}[a+b]$$

and that

$$\text{shear}[0] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

come up with the b that makes

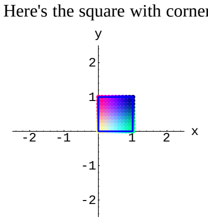
$$\text{shear}[a]^{-1} = \text{shear}[b]\,.$$

□L.16)

Given two invertible matrices A and B, the the inverse of A.B is
a) $A^{-1}.B^{-1}$
b) $B^{-1}.A^{-1}$.
My choice is ....

□L.17)

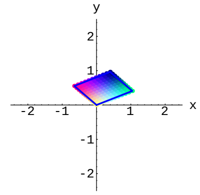
Here's the square with corners at {0,0}, {1,0}, {1,1} and {0,1} with lots of points inside:



Here's a random 2D matrix A:

$$\begin{pmatrix} 1.04 & -0.646 \\ 0.409 & 0.555 \end{pmatrix}$$

Here's what you get when you hit this square and the points inside with A:



Look at A again:

```
MatrixForm[A]
```

$$\begin{pmatrix} 1.04 & -0.646 \\ 0.409 & 0.555 \end{pmatrix}$$

Use what you see to identify the vectors that define the parallelogram.

□L.18)

Here is a cleared 2D matrix A:

```
Clear[a, b, c, d];
A = {{a, b}, {c, d}};
MatrixForm[A]
```

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Here's another 2D matrix B:

$$\begin{pmatrix} 4 & 3 \\ 2 & 2 \end{pmatrix}$$

When you calculate A.B, you get another 2D matrix

$$\begin{pmatrix} x & y \\ z & w \end{pmatrix}.$$

Your job is to write down what the values of x,y,z and w are.
x = ...
y = ...
z = ...
w = ...

□L.19)

Here are calculations of A.B and B.A for two random 2D matrices A and B:

$$\begin{pmatrix} 17.8196 & 9.38419 \\ -27.4383 & -14.0446 \end{pmatrix}$$

$$\begin{pmatrix} -0.0350128 & 4.85272 \\ -1.51471 & 3.80994 \end{pmatrix}$$

Does the outcome surprise you?
Why or why not?