

Matrices, Geometry & Mathematica

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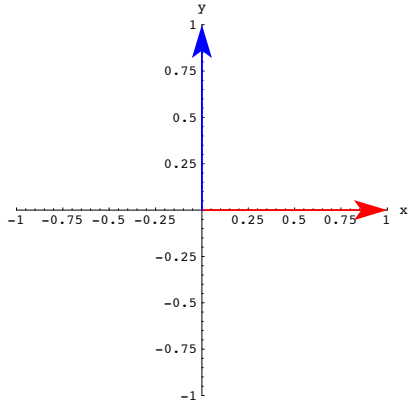
Publisher: Math Everywhere, Inc.

**MGM.03 Using Aligners, Stretchers and Hangers to Make 2D
Matrices Do What You Want
LITERACY**

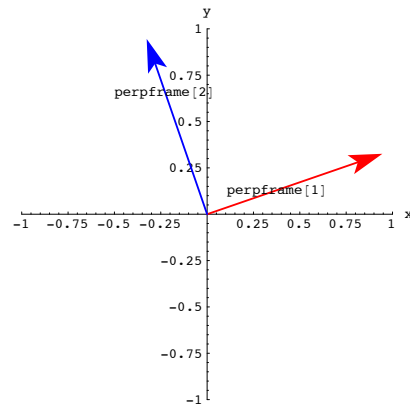
What you should be able to handle when you are away from the machine.

□L.1)

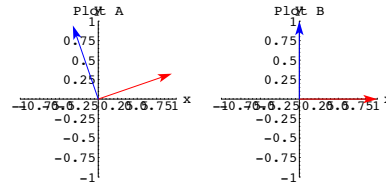
Here are the x and y axis unit vectors:



Here's a perpendicular frame:



When you hit this perpendicular frame with the aligner corresponding to the same perpendicular frame, you get one of the following outcomes:



The correct plot is Plot.....

□L.3)

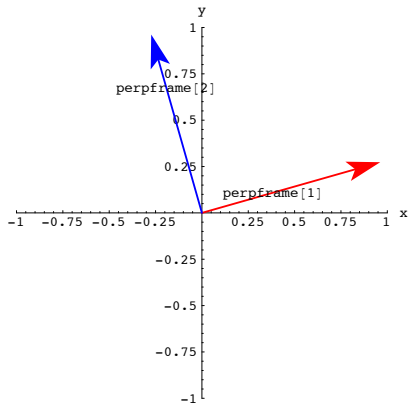
Agree or disagree:

When you hit a 2D perpendicular frame with a 2D matrix, you get another perpendicular frame.

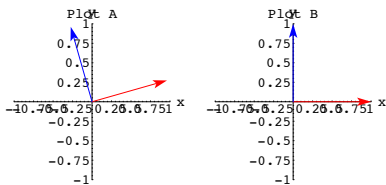
Agree..... Disagree.....

□L.4)

Here's a perpendicular frame



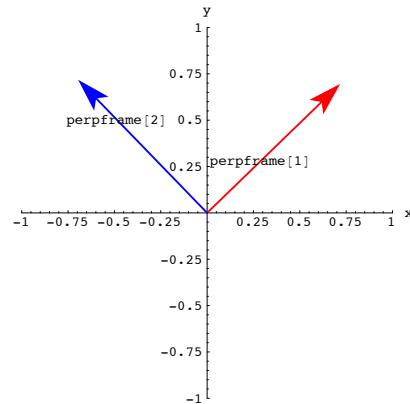
When you hit the x-y axis unit vectors with the hanger corresponding to this perpendicular frame, you get one of the following outcomes:



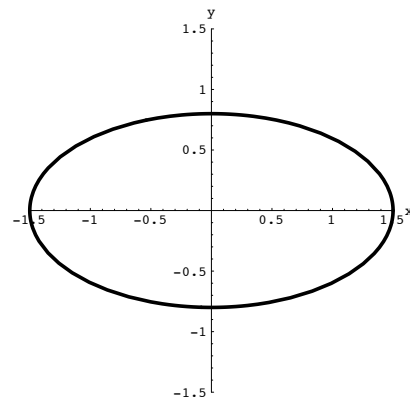
The correct plot is Plot.....

□L.2)

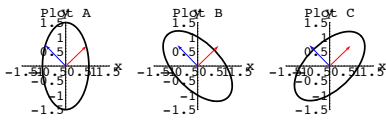
Here's a perpendicular frame:



Here is an ellipse:



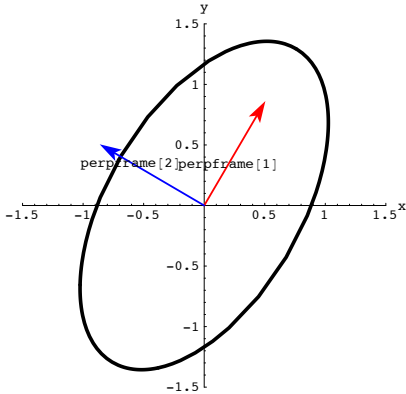
When you hit this ellipse with the hanger corresponding to the perpendicular frame plotted above, you get one of the following outcomes:



The correct plot is Plot.....

□L.5)

Here's a perpendicular frame shown with a certain ellipse:



When you hit this ellipse with the aligner corresponding to the perpendicular frame plotted above, you get one of the following outcomes:

{1.5, 0.2}

The perpendicular frame coordinates (for the perpendicular frame specified above) {u,v} of this point are given by

u =
v =

□L.9)

Here's a cleared 2D perpendicular frame:

$$\{\text{perpframe}[1], \text{perpframe}[2]\} = \{\{\cos[s], \sin[s]\}, \{-\sin[s], \cos[s]\}\}$$

Here is a point with usual xy coordinates:

{0.9, 0.8}

Fill the blank below with either the word "aligner" or "hanger."
I can calculate the perpframe coordinates of this point by hitting {x,y} with the for this perpendicular frame.

□L.10)

Here's a cleared 2D perpendicular frame:

$$\{\text{perpframe}[1], \text{perpframe}[2]\} = \{\{\cos[s], \sin[s]\}, \{-\sin[s], \cos[s]\}\}$$

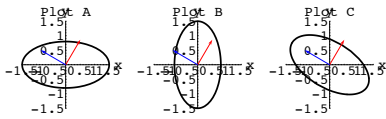
Here is a point with usual perpframe coordinates:

{0.3, 1.6}

Fill the blank below with either the word "aligner" or "hanger."
I can calculate the usual xy coordinates of this point by hitting {u,v} with the for this perpendicular frame.

□L.11)

Here is a perpendicular frame and a curve:



The correct plot is Plot.....

□L.6)

Here's a 2D perpendicular frame

$$\{\text{perpframe}[1], \text{perpframe}[2]\} = \left\{ \left\{ \frac{\sqrt{3}}{2}, \frac{1}{2} \right\}, \left\{ -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\} \right\}$$

Given that the hanger corresponding to this perpendicular frame is

$$\text{hanger} = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

fill in the appropriate values:

a = b =
c = d =

□L.7)

Here's a 2D perpendicular frame:

$$\{\text{perpframe}[1], \text{perpframe}[2]\} = \left\{ \left\{ \frac{1}{2}, \frac{\sqrt{3}}{2} \right\}, \left\{ -\frac{\sqrt{3}}{2}, \frac{1}{2} \right\} \right\}$$

Given that the aligner corresponding to this perpendicular frame is

$$\text{aligner} = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

fill in the appropriate values:

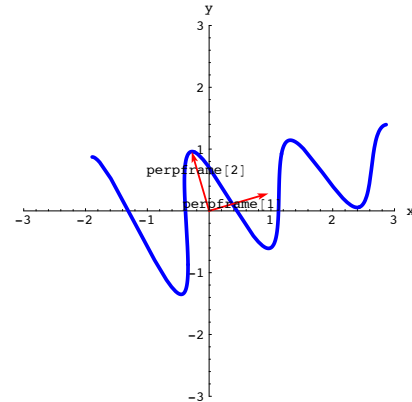
a = b =
c = d =

□L.8)

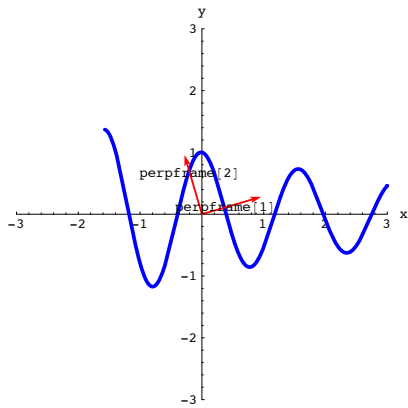
Here's a cleared 2D perpendicular frame:

$$\{\text{perpframe}[1], \text{perpframe}[2]\} = \{\{\cos[s], \sin[s]\}, \{-\sin[s], \cos[s]\}\}$$

Here is a point in usual xy coordinates:



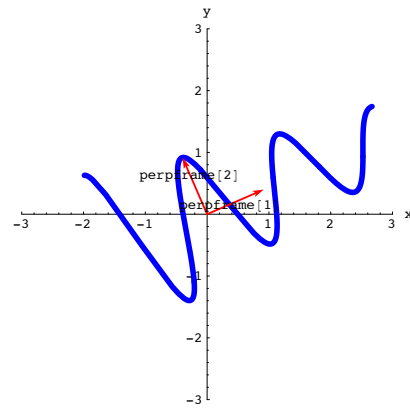
Here is what you get when you hit this curve with a certain matrix A:



Given that this matrix A is either the aligner or the hanger based on the plotted perpendicular frame, which is it:
 The aligner?
 The hanger?

□L.12)

Here is a perpendicular frame and a curve:



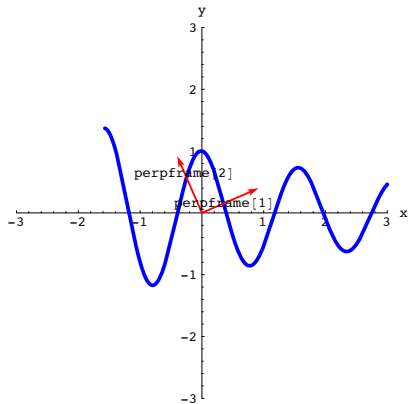
Given that this matrix A is either the aligner or the hanger based on the plotted perpendicular frame, which is it:
 The aligner?
 The hanger?

□L.13)

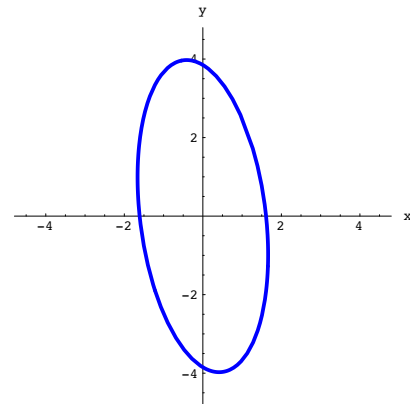
Here's a random 2D matrix
 $A = \text{hanger.stretcher.aligner}$
 made through matrix maker ingredients:

$$\begin{pmatrix} -1.21864 & 1.13069 \\ -1.89471 & -3.49379 \end{pmatrix}$$

When you hit the unit circle centered at {0,0} with A you get this ellipse:



Here is what you get when you hit this curve with a certain matrix A:



Agree or disagree with each of the following statements:

- 1) When you change the stretcher by going with new stretch factors and replot, you are guaranteed to get the same physical ellipse as plotted above.
 Agree..... Disagree.....
- 2) When you change the aligner by going with a different aligner frame and replot, you are guaranteed to get the same physical ellipse as plotted above.
 Agree..... Disagree.....
- 3) When you change the hanger by going with a different hanger frame and replot, you are guaranteed to get the same physical ellipse as plotted above.
 Agree..... Disagree.....

□L.14.i)

Here's a random 2D matrix
 $A = \text{hanger.stretcher.aligner}$
 made through matrix maker ingredients.
 The aligner is:

$$\begin{pmatrix} 0.714135 & -0.700008 \\ 0.700008 & 0.714135 \end{pmatrix}$$

The stretcher is:

$$\begin{pmatrix} 5. & 0 \\ 0 & 1.6 \end{pmatrix}$$

The hanger is:

$$\begin{pmatrix} 0.792982 & -0.609245 \\ 0.609245 & 0.792982 \end{pmatrix}$$

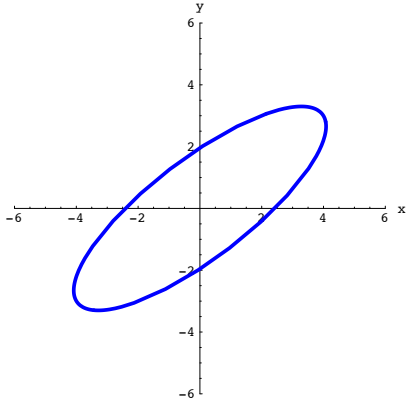
The resulting matrix

$$A = \text{hanger.stretcher.aligner}$$

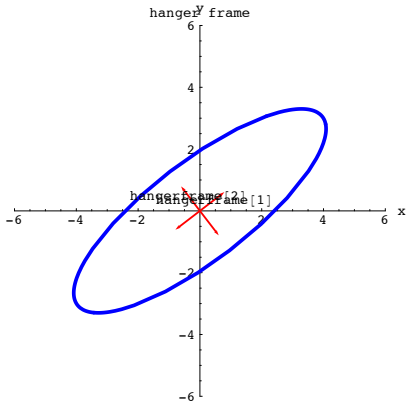
is:

$$\begin{pmatrix} 2.14912 & -3.4716 \\ 3.06357 & -1.22631 \end{pmatrix}$$

When you hit the unit circle centered at {0,0} with A you get this ellipse:



Look at this plot of the hanger frame and its negatives:



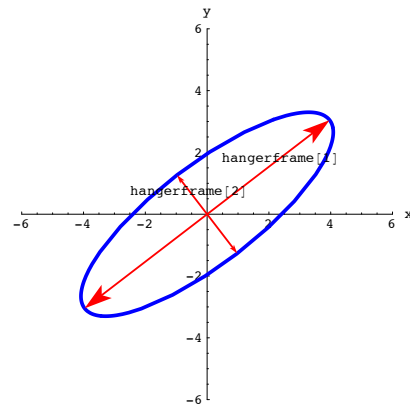
Do you think the outcome is an accident?

Yes.....

No.....

□L.14.ii)

Stay with the same set up as in part i) and look at this plot in which the unit hanger frame vectors have been multiplied by their associated stretch factors:



Do you think the outcome is an accident?

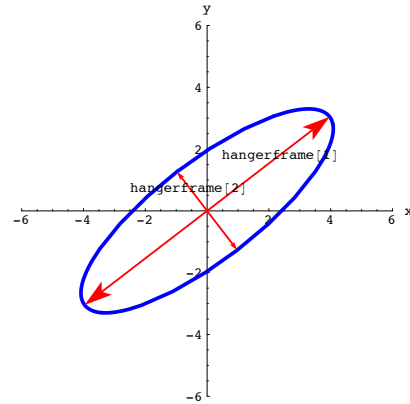
Yes.....

No.....

If not, why not?

□L.14.iii)

Here's another look at the plot in part ii) above:



And look at the stretch factors used to make A:

{5., 1.6}

Fill the blanks.

The length of the long axis of this ellipse measures out to

The length of the short axis of this ellipse measures out to

The area enclosed by this ellipse measures out to.....

□L.15)

For what kind of perpendicular frames is the corresponding hanger guaranteed to be a rotation matrix?

For what kind of perpendicular frames is the corresponding aligner guaranteed to be a rotation matrix?

□L.16)

Go with the hanger and the aligner corresponding to a given 2D perpendicular frame.

When you multiply out

hanger.aligner,

you get

$$\text{hanger.aligner} = \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}.$$

(Fill the boxes.)

□L.17)

If $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is the aligner corresponding to a given perpendicular frame, then the hanger

corresponding to the same perpendicular frame is $\text{hanger} = \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$.

(Fill the boxes.)

□L.18)

You are given a perpendicular frame {perpframe[1], perpframe[2]}. Explain how you go about making a matrix A so that

$$A \cdot \text{perpframe}[1] = 3.2 \text{ perpframe}[1]$$

and

$$A \cdot \text{perpframe}[2] = 0.5 \text{ perpframe}[2].$$

□L.19)

You are given a perpendicular frame {perpframe[1], perpframe[2]}. Explain how you go about making a matrix A so that

$$A \cdot \text{perpframe}[1] = 0.7 \text{ perpframe}[2]$$

and

$$A \cdot \text{perpframe}[2] = 1.3 \text{ perpframe}[1].$$

□L.20)

When you are making a matrix

$$A = \text{hanger.stretcher.aligner},$$

what do you do so to be sure that A is invertible?

□L.21)

You are given a line through {0, 0} in the direction of a unit vector{a, b}.

Define (by filling the boxes)

$$\text{aligner} = \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix},$$

$$\text{stretcher} = \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} \text{ and}$$

$$\text{hanger} = \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$$

so that when you hit any {x, y} with

$$A = \text{hanger.stretcher.aligner},$$

you get the point on the line closest to {x, y}.

□Tip: The vector {-b, a} is a unit vector perpendicular to the line.

□L.22)

You make a 2D matrix A by choosing a perpendicular frame to define your aligner, choosing another perpendicular frame for your hanger and two stretch factors xstretch and ystretch and then putting

$$A = \text{hanger} \cdot \begin{pmatrix} \text{xstretch} & 0 \\ 0 & \text{ystretch} \end{pmatrix} \cdot \text{aligner}.$$

Using this, you can compute A^t via the formula:

$$\text{a) } A^t = \text{aligner} \cdot \begin{pmatrix} \text{xstretch} & 0 \\ 0 & \text{ystretch} \end{pmatrix} \cdot \text{hanger}$$

$$\text{b) } A^t = \text{hanger}^t \cdot \begin{pmatrix} \text{ystretch} & 0 \\ 0 & \text{xstretch} \end{pmatrix} \cdot \text{aligner}^t$$

$$\text{c) } A^t = \text{aligner}^t \cdot \begin{pmatrix} \text{ystretch} & 0 \\ 0 & \text{xstretch} \end{pmatrix} \cdot \text{hanger}^t$$

$$\text{d) } A^t = \text{aligner}^t \cdot \begin{pmatrix} \text{xstretch} & 0 \\ 0 & \text{ystretch} \end{pmatrix} \cdot \text{hanger}^t$$

My choice is

□L.23)

You make a 2D matrix A by choosing a perpendicular frame to define your aligner, choosing another perpendicular frame for your hanger and two stretch factors xstretch and ystretch and then putting

$$A = \text{hanger} \cdot \begin{pmatrix} \text{xstretch} & 0 \\ 0 & \text{ystretch} \end{pmatrix} \cdot \text{aligner}.$$

Using this, you can compute A⁻¹ via the formula:

$$\text{a) } A^{-1} = \text{aligner} \cdot \begin{pmatrix} \frac{1}{\text{xstretch}} & 0 \\ 0 & \frac{1}{\text{ystretch}} \end{pmatrix} \cdot \text{hanger}$$

$$\text{b) } A^{-1} = \text{hanger}^t \cdot \begin{pmatrix} \frac{1}{\text{xstretch}} & 0 \\ 0 & \frac{1}{\text{ystretch}} \end{pmatrix} \cdot \text{aligner}^t$$

$$\text{c) } A^{-1} = \text{aligner}^t \cdot \begin{pmatrix} \frac{1}{\text{ystretch}} & 0 \\ 0 & \frac{1}{\text{xstretch}} \end{pmatrix} \cdot \text{hanger}^t$$

$$\text{d) } A^{-1} = \text{aligner}^t \cdot \begin{pmatrix} \frac{1}{\text{xstretch}} & 0 \\ 0 & \frac{1}{\text{ystretch}} \end{pmatrix} \cdot \text{hanger}^t$$

My choice is

□L.24)

You make a 2D matrix A by choosing a perpendicular frame to define your aligner, choosing another perpendicular frame for your hanger and two stretch factors xstretch and ystretch and the putting

$$A = \text{aligner} \cdot \begin{pmatrix} \text{xstretch} & 0 \\ 0 & \text{ystretch} \end{pmatrix} \cdot \text{hanger}.$$

If xstretch ≠ 0 and ystretch ≠ 0, then you are guaranteed that given any 2D vector Y, there is exactly one 2D vector X with A.X = Y.

Agree.....

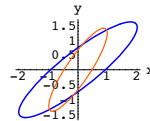
Disagree.....

□L.25.i)

Here's a random matrix made with matrix maker ingredients

$$\begin{pmatrix} 1.42462 & -1.40383 \\ 1.54555 & -0.470089 \end{pmatrix}$$

Here are the ellipses you get what you get when you hit the unit circle centered at {0, 0} with this matrix and its inverse:



Put

a = measurement of area enclosed by one of these ellipses

b = measurement of area enclosed by the other ellipse .

Without getting your hands dirty, calculate the product a times b.

□L.25.ii)

The stretch factors used to make the matrix in part i) are:

$$\{2.5, 0.6\}$$

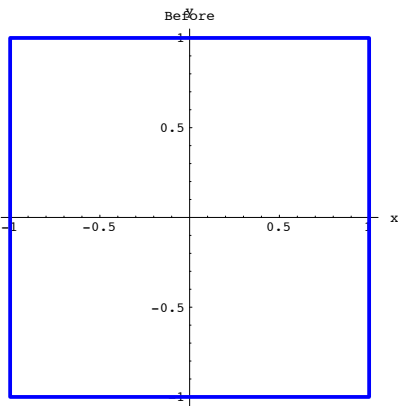
Use what you see to measure the area enclosed by each ellipse.

□L.26.i)

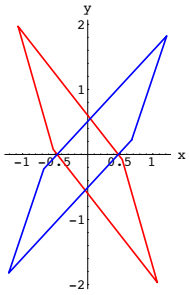
Here's a random matrix made with matrix maker ingredients

$$\begin{pmatrix} 0.269627 & 0.799854 \\ -0.944714 & -1.02228 \end{pmatrix}$$

Here is the square with corners at {-1,-1}, {1,-1}, {1,1}, and {-1,1}:



Here are the parallelograms you get what you get when you hit this square with this matrix and its transpose:



Express the measurement of area enclosed by one of these parallelograms in terms of the measurement of the area enclosed by the other.

□L.26.ii)

The stretch factors used to make the matrix in part i) are:

$$\{1.6, 0.3\}$$

Use what you see to measure the area enclosed by each parallelogram.

□L.27.i)

Here's a random 2D matrix

$A = \text{hanger.stretcher.aligner}$
made through matrix maker ingredients.

The aligner is:

$$\begin{pmatrix} 0.998339 & -0.0576085 \\ 0.0576085 & 0.998339 \end{pmatrix}$$

The stretcher is:

$$\begin{pmatrix} 4.5 & 0 \\ 0 & 1. \end{pmatrix}$$

The hanger is:

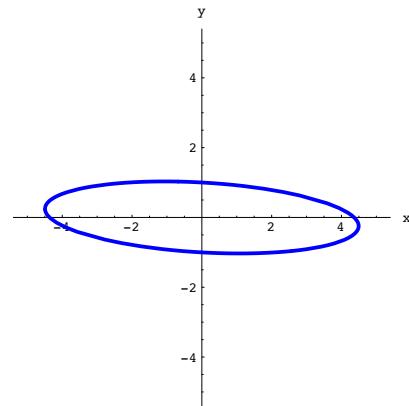
$$\begin{pmatrix} 0.582097 & -0.813119 \\ 0.813119 & 0.582097 \end{pmatrix}$$

The resulting matrix

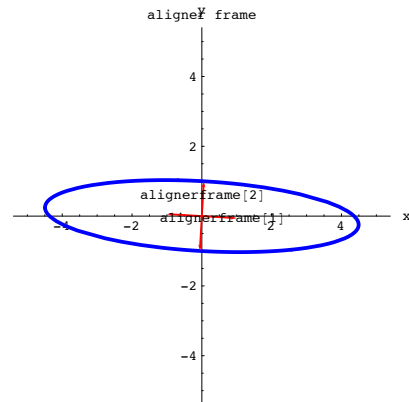
$A = \text{hanger.stretcher.aligner}$
is:

$$\begin{pmatrix} 2.56824 & -0.962671 \\ 3.68649 & 0.370339 \end{pmatrix}$$

When you hit the unit circle centered at $\{0,0\}$ with $A^!$ you get this ellipse:



Look at this plot of the aligner frame and its negatives:



Do you think the outcome is an accident?

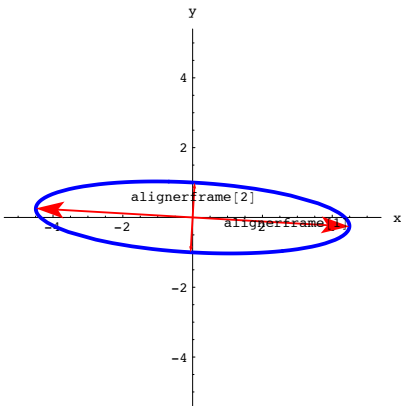
Yes.....

No.....

If not, explain your response.

□L.27.ii)

Stay with the same set up as in part i) and look at this plot in which the unit hanger frame vectors have been multiplied by their associated stretch factors:



Do you think the outcome is an accident?
 Yes.....
 No.....
 If not, explain your response.

□L.27.iii)

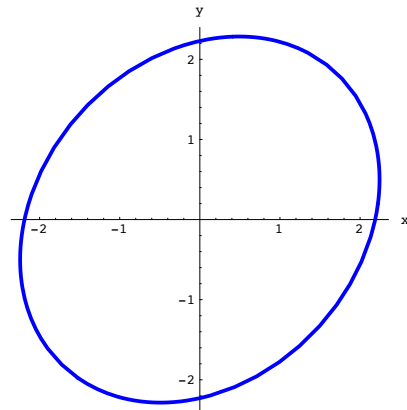
Here's another look at the plot in part ii) above:

$$\begin{pmatrix} 0.751812 & 0.659378 \\ -0.659378 & 0.751812 \end{pmatrix}$$

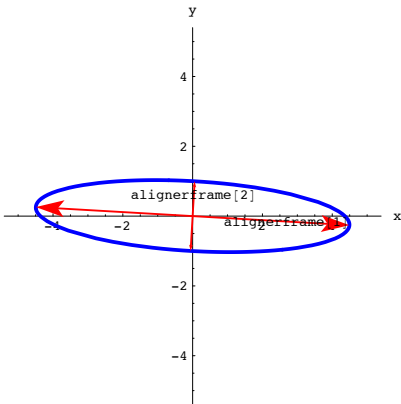
The resulting matrix
 $A = \text{hanger.stretcher.aligner}$
 is:

$$\begin{pmatrix} 0.455226 & -0.0603262 \\ -0.0390995 & 0.444524 \end{pmatrix}$$

When you hit the unit circle centered at $\{0,0\}$ with A^{-1} you get this ellipse:



Look at this plot of the aligner frame and its negatives:



And look at the stretch factors used to make A:

$$\{4.5, 1.\}$$

Fill the blanks.
 The length of the long axis of this ellipse measures out to
 The length of the short axis of this ellipse measures out to
 The area enclosed by this ellipse measures out to.....

□L.28.i)

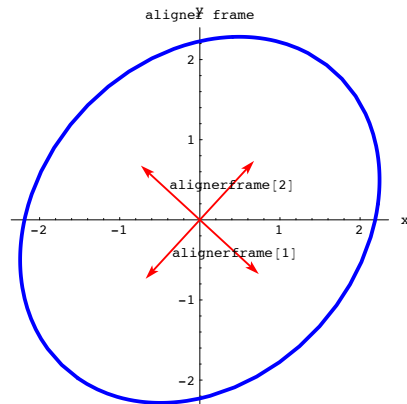
Here's a random 2D matrix
 $A = \text{hanger.stretcher.aligner}$
 made through matrix maker ingredients.
 The aligner is:

$$\begin{pmatrix} 0.736051 & -0.676926 \\ 0.676926 & 0.736051 \end{pmatrix}$$

The stretcher is:

$$\begin{pmatrix} 0.5 & 0 \\ 0 & 0.4 \end{pmatrix}$$

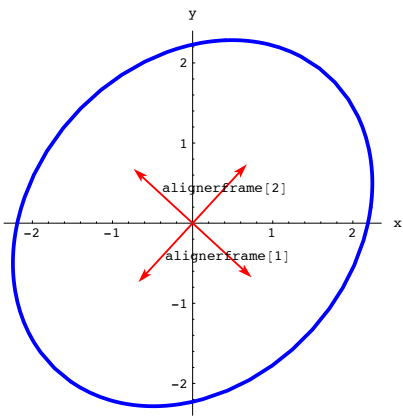
The hanger is:



Do you think the outcome is an accident?
 Yes.....
 No.....
 If not, explain your response.

□L.28.ii)

Here's another look at the plot in part i) above:



And look at the stretch factors used to make A:

$\{0.5, 0.4\}$

Fill the blanks.

The length of the long axis of this ellipse measures out to

The length of the short axis of this ellipse measures out to

The area enclosed by this ellipse measures out to.....