Matrices, Geometry&*Mathematica*

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Producer: Bruce Carpenter

Publisher: Math Everywhere, Inc.

MGM.05 3D Matrices

LITERACY

What you should be able to handle when you are away from the machine.

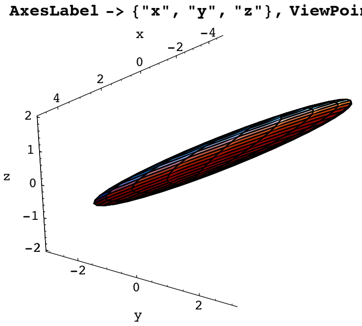
□L.1)

Here's a random 3D matrix A:

4.59756	0.933505	0.151918
-0.0906491	-2.74538	1.22645
0.657089	-1.39079	1.20612

Here's the ellipsoid you get when you hit this matrix on the 3D unit sphere centered at {0,0,0}:

```
Clear[s, t];
ParametricPlot3D[A.{Sin[s] Cos[t], Sin[s] Sin[t], Cos[s]},
  {s, 0, Pi}, {t, 0, 2 Pi}, Axes -> True,
  AxesLabel -> {"x", "y", "z"}, ViewPoint -> CMView, Boxed -> False];
```



The SVD hanger matrix for A is:

```
hanger = Transpose[SingularValues[A][[1]]];
MatrixForm[hanger]
```

-0.98191	-0.126663	-0.140745
0.183036	-0.825268	-0.534257
-0.0484815	-0.550354	0.833523

The SVD stretcher matrix for A is:

```
stretcher = DiagonalMatrix[SingularValues[A][[2]]];
MatrixForm[stretcher]
```

4.75887	0	0
0	3.4805	0
0	0	0.376371

The SVD aligner matrix for A is:

```
aligner = SingularValues[A][[3]];
MatrixForm[aligner]
```

-0.958807	-0.284037	0.00353884
-0.249723	0.836909	-0.487054
-0.13538	0.467875	0.873365

Fill the blanks:
The length of the longest axis of this ellipsoid is ....
The length of the shortest axis of this ellipsoid is....................
A unit vector that points in the direction of the longest axis of this ellipsoid is
{...................................., ..., ...}
A unit vector that points in the direction of the shortest axis of this ellipsoid is
{...................................., ..., ...}
The volume enclosed by this ellipsoid measures out to
(...............................) times (............................) times (............................) times π.

□L.2)

Given a 3D matrix A how do you use the SVD stretch factors of A to determine whether A is invertible?

□L.3)

Here's a 3D matrix A:

4.6	1.1	0.4
2.4	1.2	-1.6
1.	0.2	0.2

The non-zero SVD stretch factors for A are:

```
stretches = SingularValues[A][[2]]
```

{5.52111, 1.69923}

i) The rank of this matrix is................
When you hit this matrix on the 3D unit sphere centered at {0,0,0} you get
  a) An ellipse on a plane.
  b) An ellipsoid whose volume measures out to a positive value.
  c) A 3D line segment
ii) My choice is.................
iii) Give a general description of the 3D vectors Y for which at least one solution X of A.X = Y is guaranteed.

□L.4)

Here's another 3D matrix A:

1.4	2.8	-2.6
-0.7	-1.4	1.3
-2.8	-5.6	5.2

The non-zero SVD stretch factors for A are:

```
stretches = SingularValues[A][[2]]
```

{9.32416}

i) The rank of this matrix is................
When you hit this matrix on the 3D unit sphere centered at {0,0,0} you get
  a) An ellipse on a plane.
  b) An ellipsoid whose volume measures out to a positive value.
  c) A 3D line segment
ii) My choice is.................
iii) Give a general description of the 3D vectors Y for which at least one solution X of A.X = Y is guaranteed.

□L.5)

Here's another 3D matrix A:

1.4	2.8	-2.6
5.	-0.6	1.9
3.	-2.7	0.3

The non-zero SVD stretch factors for A are:

```
stretches = SingularValues[A][[2]]
```

{6.41532, 4.16921, 1.8361}

i) The rank of this matrix is................
ii) When you hit this matrix on the 3D unit sphere centered at {0,0,0} you get
  a) An ellipse on a plane.
  b) An ellipsoid whose volume measures out to a positive value.
  c) A 3D line segment
My choice is.................
iii) Give a general description of the 3D vectors Y for which at least one solution X of A.X = Y is guaranteed.

□L.6)

Lots of folks like to say that a given 3D matrix A is:
-> invertible if Det[A] ≠ 0
-> not invertible if Det[A] = 0.
Explain why they are right.

```
MatrixForm[DiagonalMatrix[{a, b, c}]]
```

$$\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$$

□L.7)

Given a 3D matrix A you do an SVD analysis and get

$$A = \text{aligner}.\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}.\text{hanger}.$$

Using this, you can compute A^t via the formula:

a) $A^t = \text{aligner}.\begin{pmatrix} c & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & a \end{pmatrix}.\text{hanger}$ b) $A^t = \text{aligner}^t.\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}.\text{hanger}^t$

c) $A^t = \text{aligner}^t.\begin{pmatrix} c & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & a \end{pmatrix}.\text{hanger}^t$ d) $A^t = \text{hanger}^t.\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}.\text{aligner}^t$

My choice is ....

□L.8)

Given an invertible 2D matrix A you do an SVD analysis and get

$$A = \text{aligner}.\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}.\text{hanger}.$$

Using this, you can compute A^{-1} via the formula:

a) $A^{-1} = \text{aligner}.\begin{pmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{pmatrix}.\text{hanger}$ b) $A^{-1} = \text{aligner}^t.\begin{pmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{pmatrix}.\text{hanger}^t$

c) $A^{-1} = \text{hanger}^t.\begin{pmatrix} \frac{1}{c} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{a} \end{pmatrix}.\text{aligner}^t$ d) $A^{-1} = \text{hanger}^t.\begin{pmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{pmatrix}.\text{aligner}^t$

My choice is ....

□L.9)

Given a 2D matrix A you do an SVD analysis and get

$$A = \text{aligner}.\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}.\text{hanger}.$$

with a , b and c all positive.
This information tells you that given any 3D vector Y, there is exactly one 3D vector X with A.X = Y
Agree.......................................
Disagree.......................................

□L.10)

Given a 3D matrix A you do an SVD analysis and get

$$A = \text{hanger}.\begin{pmatrix} 3.0 & 0 & 0 \\ 0 & 1.0 & 0 \\ 0 & 0 & 0.2 \end{pmatrix}.\text{aligner}.$$

You plot the hanger frame and learn it is a right hand perpendicular frame.
You plot the hanger frame and learn it is a right hand perpendicular frame.
Fill the blank:
Det[A] = ...

□L.11)

Given a 3D matrix A you do an SVD analysis and get

$$A = \text{hanger}.\begin{pmatrix} 5.0 & 0 & 0 \\ 0 & 3.0 & 0 \\ 0 & 0 & 1.0 \end{pmatrix}.\text{aligner}.$$

You plot the hanger frame and learn it is a left hand perpendicular frame.
You plot the hanger frame and learn it is a left hand perpendicular frame.
Fill the blank:
Det[A] = ...

□L.12)

Given a 3D matrix A you do an SVD analysis and get

$$A = \text{hanger}.\begin{pmatrix} 2.0 & 0 & 0 \\ 0 & 1.0 & 0 \\ 0 & 0 & 0.9 \end{pmatrix}.\text{aligner}.$$

You plot the hanger frame and learn it is a left hand perpendicular frame.
You plot the hanger frame and learn it is a right hand perpendicular frame.
Fill the blank:
Det[A] = ...

□L.13)

Given a 3D matrix A, explain why

$$\text{Det}[A^t] = \text{Det}[A].$$

□L.14)

You are given a 3D matrix A and learn that Det[A] < 0. This tells you that a hit with A incorporates a flip.
Agree......................
Disagree..................

□L.15)

You are given a 3D matrix A and learn that Det[A] > 0. This tells you that a hit with A incorporates either no flip or two flips.
Agree......................
Disagree..................

□L.16)

Given a 3D invertible matrix A, explain why

$$\text{Det}[A^{-1}] = \frac{1}{\text{Det}[A]}.$$

□L.17)

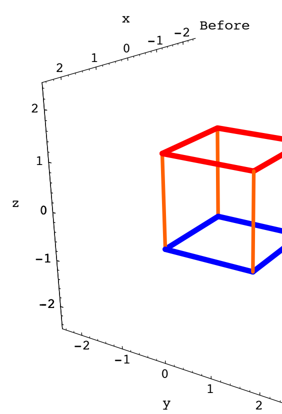
You are given a 3D matrix A and after you do your SVD analysis of it, you learn that all three SVD stretch factors are positive.
You make the call:
Can there be an {x,y,z} with
{x,y,z} ≠ {0,0,0}
and with
A.{x,y,z} = {0,0,0}?

□L.18)

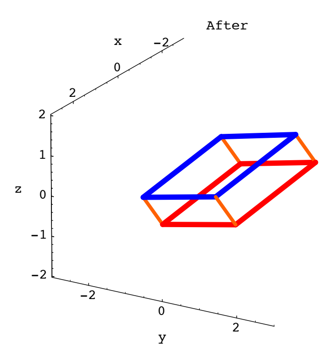
Here's a 3D matrix A:

$$\begin{pmatrix} 1.86742 & -0.913525 & -0.0726543 \\ 2.10676 & 0.529007 & 0.223607 \\ 1.03647 & 0.570634 & -0.323607 \end{pmatrix}$$

Here is the 3D cube with corners at {-1,-1,-1},
{1,-1,-1},{1,1,-1},{-1,1,-1},{-1,1,1},{-1,-1,1},{-1,-1,-1}{-1,-1,-1}{-1,-1,-1}



Here's the parallelogram-box you get when you hit this cube with A:



The determinant of A is:

```
Det[A]
-1.44
```

The area enclosed by the parallelogram-box is......................