**Matrices, Geometry&*Mathematica***

**Authors: Bruce Carpenter, Bill Davis and Jerry Uhl ©2001**

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# MGM.08 Subspaces, Spans , Dimension, Linear Independence, Basis, Orthonormal Bases

***LITERACY***

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## Fire from the hip:

## Agree, disagree or fill the blank

**□L.1)**

You are given a spanning set for a subspace S1 of kD and you are given a spanning set for a subspace S2 of kD.
Saying that S1 = S2 is the same as saying that the two spanning sets are identical.
Agree................Disagree............

**□L.2)**

Given a subspace S of 5D, it is automatic that if X and Y are in S then X + Y is also in S.
Agree............... Disagree............

**□L.3)**

Given a subspace S of 5D, it is automatic that if X is in S and t is any real number, then t X is also in S.
Agree................Disagree............

**□L.4)**

The vector {0,0,0,0,0} is in every subspace of 5D.
Agree................Disagree............

**□L.5)**

A subspace S of 5D is defined by:

```
Clear[i, spanner];
spanner[1] = {2.5, -1.8, -0.7, -3.8, 0.7};
spanner[2] = {0.5, -0.4, 2.8, 3.4, 1.7};
spanner[3] = {-5.1, -2., 0.2, -2.1, 0.0};
spanner[4] = {-3.1, -3.4, -3.3, -9.3, -1.0};
spanners = Table[spanner[i], {i, 1, 4}]
```

```
{{2.5, -1.8, -0.7, -3.8, 0.7}, {0.5, -0.4, 2.8, 3.4, 1.7},
 {-5.1, -2., 0.2, -2.1, 0}, {-3.1, -3.4, -3.3, -9.3, -1.}}
```

The corresponding SpannerMatrix is:

```
SpannerMatrix = Transpose[spanners];
MatrixForm[SpannerMatrix]
```

$$\begin{pmatrix} 2.5 & 0.5 & -5.1 & -3.1 \\ -1.8 & -0.4 & -2. & -3.4 \\ -0.7 & 2.8 & 0.2 & -3.3 \\ -3.8 & 3.4 & -2.1 & -9.3 \\ 0.7 & 1.7 & 0 & -1. \end{pmatrix}$$

The vertical columns of SpannerMatrix form a spanning set for S.
Agree................Disagree.............

**□L.6)**

Stay with the same subspace S of 5D as defined in L.5) immediately above.
The Sprojection matrix for S is:

```
rank = Length[SingularValues[SpannerMatrix][[2]]];
Sdim = rank;
Clear[Sperpframe, k];
Sperpframe[k_] := SingularValues[SpannerMatrix][[1]][[k]]
aligner = Table[Sperpframe[k], {k, 1, Sdim}];
hanger = Transpose[aligner];
Sprojection = hanger.aligner;
MatrixForm[Sprojection]
```

$$\begin{pmatrix} 0.978623 & 0.0756924 & -0.0289241 & -0.0229279 & 0.117593 \\ 0.0756924 & 0.473718 & -0.262411 & 0.292405 & -0.298697 \\ -0.0289241 & -0.262411 & 0.4455 & 0.26735 & 0.325357 \\ -0.0229279 & 0.292405 & 0.26735 & 0.802663 & 0.029877 \\ 0.117593 & -0.298697 & 0.325357 & 0.029877 & 0.299495 \end{pmatrix}$$

The vertical columns of the Sprojection matrix also form a spanning set for S.
Agree................Disagree............

**□L.7)**

Stay with the same subspace S of 5D as defined in part i) immediately above.
The hanger frame for SpannerMatrix is

```
rank = Length[SingularValues[SpannerMatrix][[2]]];
Sdim = rank;
Clear[Sperpframe, k];
Sperpframe[k_] := SingularValues[SpannerMatrix][[1]][[k]]
ColumnForm[Table[Sperpframe[k], {k, 1, Sdim}]]
```

```
{-0.297777, -0.306839, -0.303636, -0.846001, -0.0962604}
{0.93212, 0.078418, -0.214429, -0.280875, 0.0114624}
{-0.145271, 0.61108, -0.554369, 0.0897498, -0.538607}
```

These three vectors (which are guaranteed to form a perpendicular frame) matrix form an orthonormal basis for S.
Agree................Disagree............

**□L.8)**

Stay with the same subspace S of 5D as defined in part i) immediately above.
Take another look at the hanger frame for the SpannerMatrix:

```
rank = Length[SingularValues[SpannerMatrix][[2]]];
Sdim = rank;
Clear[Sperpframe, k];
Sperpframe[k_] := SingularValues[SpannerMatrix][[1]][[k]]
ColumnForm[Table[Sperpframe[k], {k, 1, Sdim}]]
```

```
{-0.297777, -0.306839, -0.303636, -0.846001, -0.0962604}
{0.93212, 0.078418, -0.214429, -0.280875, 0.0114624}
{-0.145271, 0.61108, -0.554369, 0.0897498, -0.538607}
```

The fact that the hanger frame for the SpannerMatrix consists of three vectors signals that the dimension of S is 3.
Agree................Disagree............

**□L.9)**

You are given a set of three 2D vectors X1, X2 and X3.
It is possible that
{X1, X2, X3}
turns out to be a linearly independent set.
Agree................
Disagree............

**□L.10)**

You are dealing with a subspace S of 9D and find that the dimension of S is 9.
This allows for the possibility that S is not all of 9D.
Agree................Disagree............

**□L.11)**

Given a matrix A that hits on 3D and hangs in 5D,
The column space of A, R[A], consists of all possible hits with A.
The set of horizontal row vectors of A gives you a spanning set for R[A].
Agree................Disagree............

**□L.12)**

Given a matrix A that hits on 4D and hangs in 2D,
The row space of A, R[$A^t$], consists of all possible hits with $A^t$.
The set of horizontal row vectors of A gives you a spanning set for R[$A^t$]
Agree................Disagree............

**□L.13)**

Given any matrix A, the dimension of the row space of A is the same as the dimension of the column space of A.
Agree................Disagree............

**□L.14)**

Given any matrix A, then the columns of A form a basis of the column space of A.
Agree................Disagree............

**□L.15)**

Here's a spanning set for a subspace S of 5D:

```
Clear[s, i, spanner];
spanner[1] = {3.43, -2.57, 0.19, -3.94, -2.78};
spanner[2] = {0.97, 1.29, 0.83, 5.23, 0.0};
spanner[3] = {-0.14, 1.08, 0.0, 0.0, -1.45};
spanner[4] = {0.0, 1.0, -1.10, 0.89, 1.51};
spanners = Table[spanner[i], {i, 1, 4}]
```

```
{{3.43, -2.57, 0.19, -3.94, -2.78}, {0.97, 1.29, 0.83, 5.23, 0},
 {-0.14, 1.08, 0, 0, -1.45}, {0, 1., -1.1, 0.89, 1.51}}
```

The dimension of S is:

```
SpannerMatrix = Transpose[spanners];
rank = Length[SingularValues[SpannerMatrix][[2]]]
```

**□L.18)**

You have a spanning set {spanner[1], spanner[2], spanner[3], spanner[4]} for a subpsace S of 6D.

The dimension of the subspace S12 of 6D spanned by {spanner[1], spanner[2]} turns out to be 2.

The dimension of the subspace S123 of 6D spanned by
{spanner[1], spanner[2], spanner[3]} turns out to be 3.

The dimension of the subspace S1234 of 6D spanned by
{spanner[1], spanner[2], spanner[3], spanner[4]} turns out to be 4.

This is enough to tell you that {spanner[1], spanner[2], spanner[3], spanner[4]} is an orthonormal basis for S .

**□L.19)**

You have a spanning set {spanner[1], spanner[2], spanner[3], spanner[4]} for a subpsace S of 6D.

The dimension of the subspace S12 of 6D spanned by {spanner[1], spanner[2]} turns out to be 2.

The dimension of the subspace S123 of 6D spanned by
{spanner[1], spanner[2], spanner[3]} turns out to be 2.

The dimension of the subspace S1234 of 6D spanned by
{spanner[1], spanner[2], spanner[3], spanner[4]} turns out to be 3.

This is enough to tell you that spanner[3] can be written in the form

spanner[3] = x[1] spanner[1] + x[2] spanner[2]

for the right choice of numbers x[1] and x[2].

**□L.20)**

A basis of a subspace S of kD is a linearly independent spanning set for S.
Agree................
Disagree...........

**□L.21)**

Any perpendicular frame is an orthonormal basis of the subspace it spans.
Agree................
Disagree...........



This is enough information for you to be sure that the set

{spanner[1], spanner[2], spanner[3], spanner[4]}

is linearly independent.
Agree................
Disagree............

**□L.16)**

You have a spanning set {spanner[1], spanner[2], spanner[3], spanner[4]} for a subpsace S of 6D.

The dimension of the subspace S12 of 6D spanned by {spanner[1], spanner[2]} turns out to be 2.

The dimension of the subspace S123 of 6D spanned by
{spanner[1], spanner[2], spanner[3]} turns out to be 2.
The dimension of the subspace S1234 of 6D spanned by
{spanner[1], spanner[2], spanner[3], spanner[4]} turns out to be 3.

This is enough to tell you that {spanner[1], spanner[2], spanner[3], spanner[4]} is not a basis for S.

**□L.17)**

You have a spanning set {spanner[1], spanner[2], spanner[3], spanner[4]} for a subpsace S of 6D.

The dimension of the subspace S12 of 6D spanned by {spanner[1], spanner[2]} turns out to be 2.

The dimension of the subspace S123 of 6D spanned by
{spanner[1], spanner[2], spanner[3]} turns out to be 3.

The dimension of the subspace S1234 of 6D spanned by
{spanner[1], spanner[2], spanner[3], spanner[4]} turns out to be 4.

This is enough to tell you that {spanner[1], spanner[2], spanner[4]} is a basis for S .

**□L.22)**

Inside every spanning set for a subspace S of kD is a basis of S.
Agree................
Disagree...........

**□L.23)**

You are given spanning set for a subspace S of 6D. If this spanning set consists of 6 linearly independent vectors, then it is automatic that S is all of 6D and that the given spanning set is a basis for 6D
Agree................
Disagree...........

**□L.24)**

Given a matrix A, you can be sure that the dimension of the column space of A is the same as the rank of A.
Agree................
Disagree...........

**□L.25)**

Agree or disagree:
You take the spanning set and make the Spanner Matrix and you calculate Null[SpannerMatrix].

If Null[SpannerMatrix] consists of nothing other than the vector of all zeroes, then the given spanning set is guaranteed to be linearly independent.

If Null[SpannerMatrix] contains vectors other than the vector of all zeroes, then the given spanning set is guaranteed not to be linearly independent.
Agree................
Disagree...........

**□L.26)**

If {X[1],X[2],X[3],. . .,X[k]} is an orthonormal basis of a k dimensional subspace S of nD, then given any Y in nD the vector in S closest to Y is

$$\sum_{j=1}^{k} (Y \cdot X[j])\, X[j].$$

Agree................
Disagree...........

**☐L.27)**

You are given a subspace S of 13D and a vector X in 13D.
If ||Sprojection.X|| = ||X||, then Sprojection.X = X and so X is in S.
Agree................
Disagree............