Matrices, Geometry&*Mathematica*

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MGM.09 Eigensense: Diagonalizable Matrices, Matrix Exponential, Matrix Powers and Dynamical Systems
LITERACY

In each of the following problems, indicate whether you agree or disagree.
You are not asked to give any reasons for your response.

□L.a)

If A is a diagonalizable 3D matrix and the eigenvalues of A are
$0.15, -0.52$ and 2.74,
then the eigenvalues of $E^{A\,t}$ are
$E^{0.15\,t}$, $E^{-0.52\,t}$ and $E^{2.74\,t}$.
Agree:....... Disagree:....

□L.b)

If A is a diagonalizable 3D matrix and the eigenvalues of A are
$0.15, -0.52$ and 2.74,
then the eigenvalues of A^3 are
$(0.15)^3$, $(-0.52)^3$ and $(2.74)^3$.
Agree:....... Disagree:....

□L.c)

If A is a diagonalizable 3D matrix and the eigenvalues of A are
$-0.072, -0.023$ and -0.914,
Then you are guaranteed that as t gets large
$E^{A\,t}.X \to \{0, 0, 0\}$
Agree:....... Disagree:....

□L.d)

If A is a diagonalizable 2D matrix and the eigenvalues of A are
0.52 and -2.74,
then you are guaranteed that as t gets large
$E^{A\,t}.X \to \{0, 0\}$.
Agree:....... Disagree:....

□L.e)

If A is a diagonalizable 2D matrix and the eigenvalues of A are
$-0.52 + I\,3.84$ and $-0.52 - I\,3.84$,
then you are guaranteed that as t gets large
$E^{A\,t}.X \to \{0, 0\}$.
Agree:....... Disagree:....

□L.f)

If A is a diagonalizable 2D matrix and the eigenvalues of A are
$0.52 + I\,3.84$ and $0.52 - I\,3.84$,
then you are guaranteed that as t gets large
$E^{A\,t}.X \to \{0, 0\}$.
Agree:....... Disagree:....

□L.g)

If A is a diagonalizable 3D matrix and the eigenvalues of A are
$-0.34, -0.52 + I\,0.67$ and $-0.52 - I\,0.67$,
then you are guaranteed that as t gets large
$E^{A\,t}.X \to \{0, 0, 0\}$.
Agree:....... Disagree:....

□L.h)

If A is a diagonalizable 3D matrix and the eigenvalues of A are
$0.83, I\,2.5$ and $-I\,2.5$,
then you are guaranteed that as t advances,
$E^{A\,t}.X$ spirals away from $\{0, 0, 0\}$ (provided $X \neq \{0, 0, 0\}$).
Agree:....... Disagree:....

□L.i)

If A is a diagonalizable matrix then the eigenvectors of A are the same as the eigenvectors of $E^{A\,t}$ provided $t \neq 0$.
Agree:....... Disagree:....

□L.j)

A diagonalizable 2D matrix A calculates out to have
eigenvalue[1] = 1.5 with corresponding eigenvector[1] = {0.25, 1.0}
and
eigenvalue[2] = −2.0 with corresponding eigenvector[2] = {0.5, 1.0}.
so that
A.eigenvector[1] = 1.5 eigenvector[1]
and
A.eigenvector[2] = −2.0 eigenvector[2]
Put
$\{x[t], y[t]\} = E^{A\,t}.\text{starter}$
so that {x[0], y[0]} = starter.
If starter is not a multiple of either eigenvector[1] or eigenvector[2], then as t gets large
$\frac{y[t]}{x[t]} \to 4.0$.
Agree:....... Disagree:....

□L.k)

A diagonalizable 2D matrix A calculates out to have
eigenvalue[1] = 1.5 with corresponding eigenvector[1] = {0.25, 1.0}
and
eigenvalue[2] = −2.0 with corresponding eigenvector[2] = {0.5, 1.0}.
so that
A.eigenvector[1] = 1.5 eigenvector[1]
and
A.eigenvector[2] = −2.0 eigenvector[2]
Put
$\{x[k], y[k]\} = A^k.\text{starter}$
so that {x[0], y[0]} = starter.

If starter is not a multiple of either eigenvector[1] or eigenvector[2], then as t gets large
$\frac{y[t]}{x[t]} \to 4.0$.
Agree:....... Disagree:....

□L.l)

Here's a random 2D matrix A:

```
Clear[i, j, t];
A = Table[Random[Real, {-5, 5}], {i, 1, 2}, {j, 1, 2}];
MatrixForm[A]
```

$$\begin{pmatrix} -2.04185 & 0.925096 \\ 0.276589 & 2.25256 \end{pmatrix}$$

Here are two calculations - one of which is a correct calculation of $E^{A\,t}$:

```
calculation1 = MatrixForm[Simplify[ComplexExpand[E^(A t)]]]
```

$$\begin{pmatrix} e^{-2.04185\,t} & e^{0.925096\,t} \\ e^{0.276589\,t} & e^{2.25256\,t} \end{pmatrix}$$

```
calculation2 = MatrixForm[Simplify[ComplexExpand[MatrixExp[A t]]]]
```

$$\begin{pmatrix} 0.986678\,e^{-2.10063\,t} + 0.0133223\,e^{2.31134\,t} & -0.209679\,e^{-2.10063\,t} + 0.209679\,e^{2} \\ -0.0626905\,e^{-2.10063\,t} + 0.0626905\,e^{2.31134\,t} & 0.0133223\,e^{-2.10063\,t} + 0.986678\,e^{2} \end{pmatrix}$$

The first calculation of $E^{A\,t}$ is correct and the second is wrong.
Agree:....... Disagree:....

□L.m)

Here's a 3D matrix A:

```
Clear[i, j];
A = {{1.0, 2.0, 3.0}, {4.0, 5.0, 6.0}, {7.0, 8.0, 9.0}};
MatrixForm[A]
```

$$\begin{pmatrix} 1. & 2. & 3. \\ 4. & 5. & 6. \\ 7. & 8. & 9. \end{pmatrix}$$

Here are two calculations - one of which is correct::

```
calculation1 = A^2;
MatrixForm[calculation1]
```

$$\begin{pmatrix} 1. & 4. & 9. \\ 16. & 25. & 36. \\ 49. & 64. & 81. \end{pmatrix}$$

```
calculation2 = MatrixPower[A, 2];
MatrixForm[calculation2]
```

$$\begin{pmatrix} 30. & 36. & 42. \\ 66. & 81. & 96. \\ 102. & 126. & 150. \end{pmatrix}$$

The second calculation is a correct calculation of A.A.
Agree:....... Disagree:....

□L.n)

Although the theory of diagonalizable matrices was developed in the Basics in the setting of 2D matrices, the same theory works in the higher dimensions.
Agree:....... Disagree:....

□L.o)

If A is a diagonalizable 3D matrix, then your are guaranteed that the eigenvectors of A span all of 3D.
Agree:....... Disagree:....

□L.p)

If A is a 4D matrix, and the eigenvectors of A do not span all of 4D, then your are guaranteed that A is not diagonalizable.
Agree:....... Disagree:....

□L.q)

If A is a diagonalizable matrix and none of the eigenvalues of A is zero, then A is invertible.
Agree:....... Disagree:......