Differential Equations&*Mathematica*

Produced by Bruce Carpenter **Published by Math Everywhere, Inc.**

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DE.01 Transition from Calculus to DiffEq: The Exponential Differential Equation

$y'[t] + r\,y[t] = f[t]$

Literacy Sheet

What you need to know when you're away from the machine.

□L.1)

Here are three diffeq's:
a) $y'[t] + 0.2\,y[t] = 0$
b) $y'[t] - 0.2\,y[t] = 0$
c) $y'[t] + 0.1\,y[t] = 0$.
Which of these diffeqs is solved by
$y[t] = 13\,E^{-0.2\,t}$?

□L.2)

Here are three diffeq's:
a) $y'[t] + y[t] = E^{2\,t}$
b) $y'[t] - y[t] = E^{2\,t}$
c) $y'[t] = E^{2\,t}$.
Which of these diffeqs is solved by
$y[t] = 4\,E^{t} + E^{2\,t}$?

□L.3)

Here are three diffeq's:
a) $y'[t] + 2\,t\,y[t] = 0$
b) $y'[t] - 2\,t\,y[t] = 0$
c) $y'[t] + y[t] = 0$.
Which of these diffeqs is solved by
$y[t] = 8\,E^{-t^2}$?

□L.4)

Here are three diffeq's:
a) $y''[t] + 4\,y[t] = 0$
b) $y''[t] + 9\,y[t] = 0$
c) $y''[t] + 16\,y[t] = 0$.
Which of these diffeqs is solved by
$y[t] = 5\,\mathrm{Sin}[3\,t]$?

□L.5)

Just to see that you can handle yourself away from the machine, come up with formulas for the solutions of the following diffeqs:
a) $y'[t] + 0.3\,y[t] = 5\,E^{-t}$ with $y[0] = 5$.
b) $y'[t] - 0.3\,y[t] = 5$ with $y[0] = 1$.
c) $y'[t] + y[t] = \mathrm{DiracDelta}[t - 3]$ with $y[0] = 2$.

□L.6)

You know that $y'[t] = a\,y[t]$ for all x's and you know that a is positive and that $y[0]$ is positive. Does $y[t]$ go up or down as t advances from left to right?

□L.7)

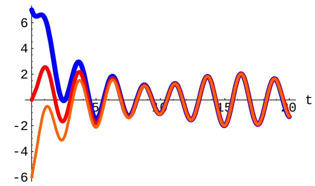
If a is negative , does the solution of $y'[t] = a\,y[t]$ with $y[0] = 10$ go up or down as t advances from left to right. Can the soution ever go negative?
Why or why not?

□L.8)

The solution y[t] of
$y'[t] + r\,y[t] = f[t]$ with $y[0] = \text{starter}$
is given by
$$y[t] = E^{-r\,t}\,\text{starter} + E^{-r\,t}\int_0^t E^{r\,t}\,f[t]\,dt.$$
Explain this:
If $r > 0$, then, no matter what starter is, the plot of y[t] will eventually merge with the plot of
$$y0[t] = E^{-r\,t}\int_0^t E^{r\,t}\,f[t]\,dt.$$

□L.9)

Here are three plots of solutions of a certain forced exponential diffeq:



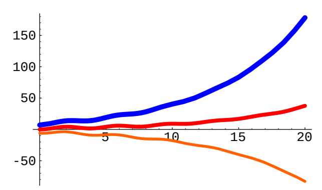
You are given that all three plots are all either solutions of
a) $y'[t] + 0.6\,y[t] = 4\,\mathrm{Sin}[2.5\,t] + \mathrm{Cos}[2\,t]$
or solutions of
b) $y'[t] - 0.6\,y[t] = 4\,\mathrm{Sin}[2.5\,t] + \mathrm{Cos}[2\,t]$.
Make your choice and say how you arrived at it.

□L.10)

The solution y[t] of
$y'[t] + r\,y[t] = f[t]$ with $y[0] = \text{starter}$
is given by
$$y[t] = E^{-r\,t}\,\text{starter} + E^{-r\,t}\int_0^t E^{r\,t}\,f[t]\,dt.$$
Explain this:
If $r < 0$ then, unless starter = 0, the plot of y[t] will NOT eventually merge with the plot of
$$y0[t] = E^{-r\,t}\int_0^t E^{r\,t}\,f[t]\,dt.$$

□L.11)

Here are three plots of solutions of a certain forced exponential diffeq:



You are given that all three plots are all either solutions of
a) $y'[t] + 0.15\,y[t] = \mathrm{Sin}[1.5\,t] + 2\,\mathrm{Sin}[1.7\,t]$
or solutions of
b) $y'[t] - 0.15\,y[t] = \mathrm{Sin}[1.5\,t] + 2\,\mathrm{Sin}[1.7\,t]$.
Make your choice and say how you arrived at it.

□**L.12)**

Here are plots of the solutions of

diffeq a): $y'[t] + 0.01\, y[t] = 0$

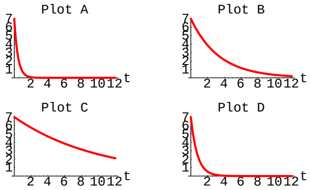
diffeq b): $y'[t] + 0.3\, y[t] = 0$,

diffeq c): $y'[t] + 1.3\, y[t] = 0$,

diffeq d): $y'[t] + 2.3\, y[t] = 0$,

all with the same starter y[0] = 7.

The plots are not in order. Your job is to match the differential equation with the plot of its solution.



diffeq a) ------> Plot....... diffeq b) ------> Plot.......

diffeq c) ------> Plot....... diffeq d) ------> Plot.......

□**L.13)**

Do you expect solutions of

$y'[t] - 0.6\, y[t] = 0$

to decay to 0 as t gets large?

Why or why not?

□**L.14)**

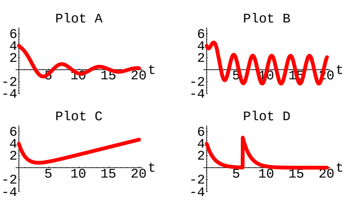
Here are plots of solutions of the forced exponential diffeq

$y'[t] + 0.8\, y[t] = f[t]$

with

$y[0] = 4.0$

for five choices of forcing functions f[t]:



The four forcing functions f[t] used in these plots are:

$f1[t] = 0.2\, t$, :

$f2[t] = 5\, \text{DiracDelta}[t - 6]$,

$f3[t] = 2.3\, E^{-0.1\, t}\, \text{Cos}[t]$,

$f4[t] = 5\, \text{Sin}[2\, t]$.

Your job is to match the plots to the forcing functions:

f1[t]--------------> Plot..... f2[t]--------------> Plot.....

f3[t]--------------> Plot..... f4[t]--------------> Plot.....

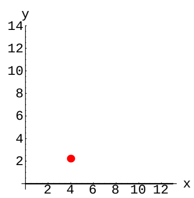
□**L.15)**

Give the formula for the solution of

$y'[x] = 0.2\, y[x]$ with $y[0] = 1$

and pencil in a rough sketch of the solution on the axes below.

The plotted point is on the curve.



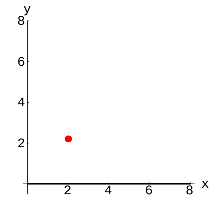
□**L.16)**

Give the formula for the solution of

$y'[x] = -0.5\, y[x]$ with $y[0] = 6$

and pencil in a rough sketch of the solution on the axes below.

The plotted point is on the curve.



□**L.17)**

You are given that

$y[t] = 37.3\, E^{-0.045\, t}$

Write down a differential equation that y[t] solves.

Include starter data.

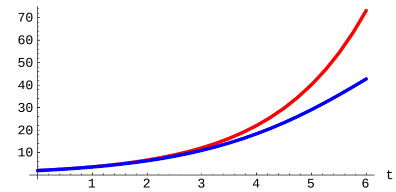
□**L.18)**

Here are plots of solutions of

$y'[t] = 0.6\, y[t]$ with $y[0] = 2$

and

$y'[t] = 0.6\, y[t]\left(1 - \frac{y[t]}{100}\right)$ with $y[0] = 2$:



Explain why as t advances from 0, the plots of both of these solutions had no choice but to share a lot of ink initially.

Explain why for larger t's, the plots had no choice but to pull apart, with one plot eventually sailing way above the other.

□**L.19)**

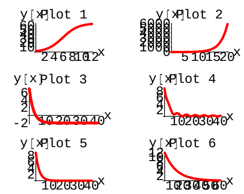
Here are six plots: They are plots of:

a. a solution of $y'[x] = r\, y[x]$ with $r > 0$.

b. a solution of $y'[x] = r\, y[x]$ with $r < 0$.

c. none of the above

Which is which?



Plot 1 ---->........... Plot 2 ---->........... Plot 3 ---->...........

Plot 4 ---->........... Plot 5 ---->........... Plot 6 ---->...........

□**L.20)**

a) You are given a solution y[t] of

$y'[t] + 5\, y[t] = 6\, \text{DiracDelta}[t - 4.7]$

What happens to y[t] at t = 4.7?

b) How does your response indicate that

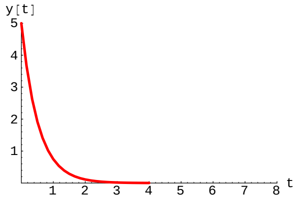
$\text{DiracDelta}[t - 4.7] \neq 2\, \text{DiracDelta}[t - 4.7]$?

□**L.21)**

Here is the part of the plot of the solution of

$y'[t] + 1.9\, y[t] = 5\, \text{DiracDelta}[t - 3]$ with $y[0] = 5$.

Your job is to sketch in the rest of the plot.



L.22)

a) Write down the values of

$\int_{0}^{4.99999} \text{DiracDelta}[t-5]\, dt$

$\int_{0}^{5.00001} \text{DiracDelta}[t-5]\, dt$

$\int_{5.00001}^{100} \text{DiracDelta}[t-5]\, dt.$

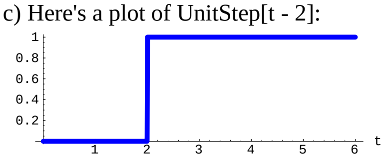
b) Write down the values of

$\int_{0}^{2.99999} E^{t}\, \text{DiracDelta}[t-3]\, dt$

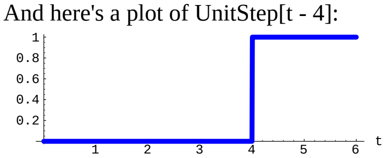
$\int_{2.999999}^{3.000001} E^{t}\, \text{DiracDelta}[t-3]\, dt$

$\int_{3.00001}^{100000} E^{t}\, \text{DiracDelta}[t-3]\, dt$

c) Here's a plot of UnitStep[t - 2]:



And here's a plot of UnitStep[t - 4]:



Digest the two plots and explain the result of this *Mathematica* calculation:

```
Clear[t, x]
∫_0^t Sin[x] DiracDelta[x - 5] dx
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Sin[5] UnitStep[-5 + t]
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