Differential Equations&*Mathematica*

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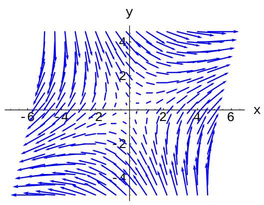
DE.06 Systems and Flows
Literacy Sheet

What you need to know when you're away from the machine.

□L.1)

Here's a rather detailed plot of a certain vector field:



Visible in this plot are four straight line trajectories and four other families of trajectories.
Pencil in the straight line trajectories and one trajectory of each of the four other families.

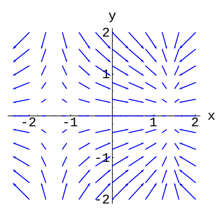
□L.2)

You are given a system of differential equations

$x'[t] = 2 - x[t]^2$

$y'[t] = -y[t]$.

You decide to look at the flow of the trajectories and you get:



To get this plot, you plotted a vector field

Field[x, y] = {m[x, y], n[x, y]}

with tails at {x, y} for a selection of points {x, y}.
The question here:
What are the exact formulas for m[x, y] and n[x, y] ?

□L.3)

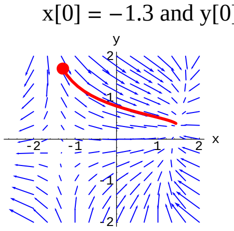
Here is the flow plot of a certain system

$x'[t] = m[x[t], y[t]]$

$y'[t] = n[x[t], y[t]]$

together with a trajectory resulting from starter data

$x[0] = -1.3$ and $y[0] = 1.7$:



Which way is the trajectory going?
In what sense does the flow plot guide the trajectory along its way?

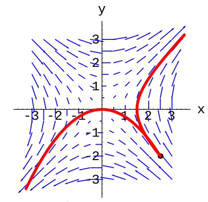
□L.4)

Here is the flow plot of a certain system

$x'[t] = m[x[t], y[t]]$

$y'[t] = n[x[t], y[t]]$

of differential equations shown with two curves:



One of these curves is a genuine trajectory.
The other is bogus.
Identify the trajectory and discuss the information that led to your choice.

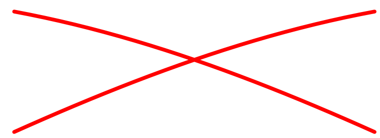
□L.5)

Can two trajectories of a system

$x'[t] = m[x[t], y[t]]$

$y'[t] = n[x[t], y[t]]$

ever cross each other like this?



Why or why not?

□L.6)

The system of differential equations is

$x'[t] = x[t]^2 + 3\,y[t]$

$y'[t] = x[t] - y[t]^2$

You are given that a trajectory {x[t], y[t]} passes through {2, 1} at a certain time $t = t^*$.
Your job is to write down the vector

$\{x'[t^*], y'[[t^*]\}$

which is tangent to the trajectory at {2, 1}.

□L.7)

a) Explain this:
When you convert the single second order differential equation

$y''[t] + 2\,y'[t] + 8\,y[t] = 0$

into a system of two first order differential equations by putting $x[t] = y'[t]$, you get

$x'[t] = -2\,x[t] - 8\,y[t]$

$y'[t] = x[t]$

b) Are plots of solutions

$y''[t] + 2\,y'[t] + 8\,y[t] = 0$

the same as plots of trajectories of the system

$x'[t] = -2\,x[t] - 8\,y[t]$

$y'[t] = x[t]$?

□L.8)

a) Convert the single second order ordinary oscillator differential equation

$y''[t] + 1.3\,y'[t] + 8.2\,y[t] = 0$

into a system of two first order differential equations by putting $x[t] = y'[t]$.

b) Explain why you need starter data on both y[0] and y′[0] to nail down and plot a specific solution of

$y''[t] + 1.3\,y'[t] + 8.2\,y[t] = 0$.

□L.9)

a) Convert the pendulum oscillator differential equation

$y''[t] + 0.2\,y'[t] + 6.5\,\mathrm{Sin}[y[t]] = 0$.

into a system of two first order differential equations by putting $x[t] = y'[t]$.

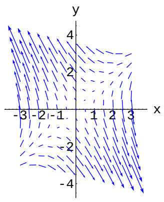
b) Explain why you need starter data on both y[0] and y′[0] to nail down and plot a specific solution of

$y''[t] + 0.2\,y'[t] + 6.5\,\mathrm{Sin}[y[t]] = 0$.

□L.10)

Here is the flow plot for the system

$x'[t] = 0.5\, x[t] - 1.7\, y[t]$
$y'[t] = -2.8\, x[t] + 2.3\, y[t].$



Shade in regions consisting of points {a, b} in which you think the behavior of the trajectories is extremely sensitive to errors in the starting data {x[0], y[0]} = {a, b}.

□L.11)

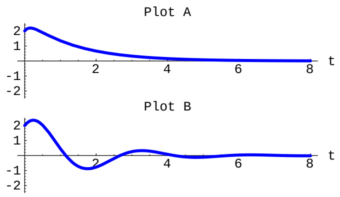
Here are plots of the solutions of

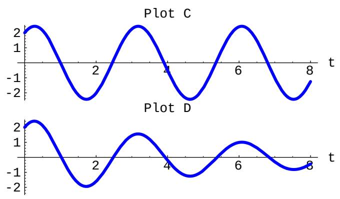
$y''[t] + 4.7\, y[t] = 0$
$y''[t] + 0.3\, y'[t] + 4.7\, y[t] = 0,$
$y''[t] + 1.3\, y'[t] + 4.7\, y[t] = 0,$
$y''[t] + 7.3\, y'[t] + 4.7\, y[t] = 0,$

all with the same starter data

$y[0] = 2$ and $y'[0] = 3.$

The plots are not in order. Your job is to match the differential equation with the plot of its solution.





How does the starter data signal that each plot must go up before it can go down?

□L.12)

Here are three oscillator differential equations:

$y''[t] + 0.1\, y'[t] + 5.8\, y[t] = 0,$
$y''[t] + 5.8\, y[t] = 0,$
$y''[t] + 1.4\, y'[t] + 5.8\, y[t] = 0.$

One is a differential equation of an undamped oscillator; the other two are differential equations of damped oscillators. Which is which?
Of the two damped oscillator differential equations, which one is the more heavily damped?

□L.13)

An ordinary damped oscillator comes from the differential equation

$y''[t] + 0.4\, y'[t] + 6.2\, y[t] = 0$

with

$y[0] = 2.7$ and $y'[0] = -1.2.$

Putting

$x[t] = y'[t]$

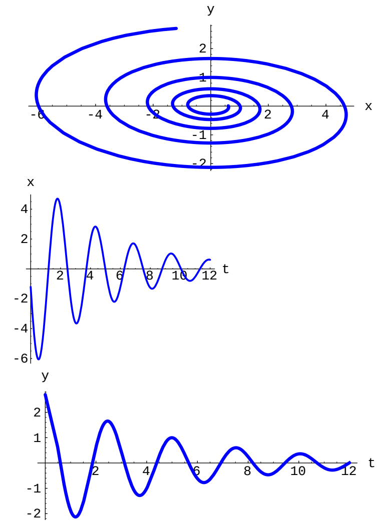
gives the equivalent linear system

$x'[t] = -0.4\, x[t] - 6.2\, y[t]$
$y'[t] = x[t]$

with

$x[0] = -1.2$ and $y[0] = 2.7.$

You use the machine to solve this system numerically and you get three plots as follows:



One of these plots is a plot of the solution of

$y''[t] + 0.4\, y'[t] + 6.2\, y[t] = 0$

with

$y[0] = 2.7$ and $y'[0] = -1.2.$

Another is a plot of the derivative of the solution of

$y''[t] + 0.4\, y'[t] + 6.2\, y[t] = 0$

with

$y[0] = 2.7$ and $y'[0] = -1.2.$

Yet another is a plot of a trajectory in the equivalent system

$x'[t] = -0.4\, x[t] - 6.2\, y[t]$
$y'[t] = x[t]$

with

$x[0] = -1.2$ and $y[0] = 2.7.$

Which is which?

□L.14)

An ordinary damped oscillator comes from the differential equation

$y''[t] + b\, y'[t] + c\, y[t] = 0$

with

$y[0] = d$ and $y'[0] = v.$

So far the exact values of the constants b, c, d and v are not known.

You are given that when this differential equation is converted to an equivalent system by putting

$x[t] = y'[t],$

the resulting system is

$x'[t] = -1.6\, x[t] - 2.3\, y[t]$
$y'[t] = x[t]$

with

$x[0] = 2.2$ and $y[0] = 3.9.$

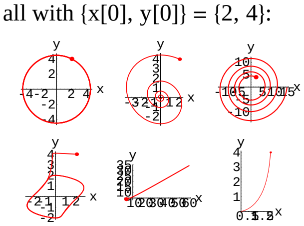
What are the exact values of b, c, d, and v?

□L.15)

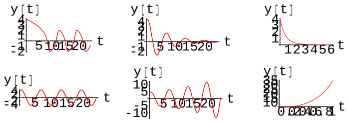
Here are six plots of trajectories for six different systems of the form

$x'[t] = m[x[t], y[t]]$
$y'[t] = n[x[t], y[t]]$

all with {x[0], y[0]} = {2, 4}:



Here are the plots of solutions, y[t], of the same six systems, all with the same starter data {x[0], y[0]} = {2, 4}:



Your job is to pair the solution plots with the corresponding trajectory plots.

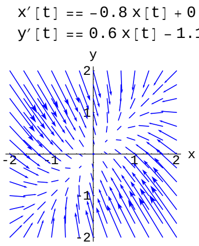
□L.16)

You are using the computer to get the flow plot for the system

$x'[t] = -0.8\, x[t] + 0.3\, y[t]$

$y'[t] = \ \ 0.6\, x[t] - 1.1\, y[t]$

and you get this:



Use the flow plot as best you can to determine whether you agree or disagree with the following statements:

a) If the given starter data on {x[0], y[0]} is a point in the plotted region and {x[t], y[t]} is the corresponding solution of this system, then you can expect that

$x[t] \to 0$ and $y[t] \to 0$

as $t \to \infty$.

b) If the given starter data on {x[0], y[0]} is a point in the plotted region and {x[t], y[t]} is the corresponding solution of this system, then you can expect that the plots of both x[t] and y[t] oscillate above and below 0 as t advances.

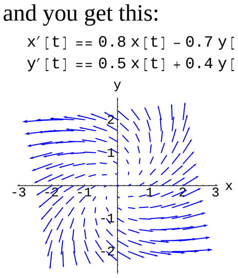
□L.17)

You are using the computer to get the flow plot for the system

$x'[t] = \ \ 0.8\, x[t] - 0.7\, y[t]$

$y'[t] = -0.5\, x[t] - 0.4\, y[t]$

and you get this:



Use the flow plot as best you can to determine whether you agree or disagree with the following statements:

a) If the given starter data on {x[0], y[0]} is a point in the plotted region and {x[t], y[t]} is the corresponding solution of this system, then you can expect that

$x[t] \to 0$ and $y[t] \to 0$

as $t \to \infty$.

b) If the given starter data on {x[0], y[0]} is a point in the plotted region and {x[t], y[t]} is the corresponding solution of this system, then you can expect that the plots of both x[t] and y[t] oscillate above and below 0 as t advances.

□LT.18)

Here is a system of differential equations:

$x'[t] = a\, x[t] + b\, y[t]$

$y'[t] = c\, x[t] + d\, y[t].$

Here a, b, c, and d stand for numerical constants.

Write down the little matrix A that allows you to express this system in the form

$\{x'[t], y'[t]\} = A . \{x[t], y[t]\}.$