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The Journal of the Elisha Mitchell Scientific Society

Volume 94
Number 2
Summer 1978

Official Publication of the Elisha Mitchell Scientific Society
and the North Carolina Academy of Science
Published at
The University of North Carolina at Chapel Hill

Vincent's Theorem and Akritas' Polynomial Real Root Isolation Algorithm. *A. G. Akritas.* In order to solve numerically an algebraic equation of degree higher than four, Fourier suggested first isolating its roots and then approximating them to any desired degree of accuracy.

Isolation of the real roots of a polynomial equation is the process of finding real intervals, such that each contains exactly one real root and every real root is contained in some interval. Since 1830 the only method widely known and used for this purpose was that of Sturm. It was implemented within a computer algebra system (using infinite-precision arithmetic) and proven to be $O(n^{13}L(|P|_\infty)^3)$, where n is the degree of the integral, univariate, square-free polynomial P, and $L(|P|_\infty)$ the length, in bits, of its maximum coefficient in absolute value.

Quite recently the author came across Vincent's remarkable theorem (1836) which asserts that, if a univariate, square-free polynomial with rational coefficients is successively transformed by successive substitutions of the form $x \leftarrow a_i + 1/x$, for arbitrary integral. positive a_i's, one eventually obtains a polynomial with at most one sign variation.

Clearly, the calculation of the quantities $a_1, a_2, \ldots$ (for a particular root) constitutes the real root isolation procedure. Two algorithms result--Vincent's and Akritas'--corresponding to the two different ways of computing these a_i's; the difference between these methods can be thought of as being analogous to the one between the integrals of Riemann and Lebesgue. Vincent's approach is by "brute force," in the sense that he computes a particular a_i by always incrementing it by 1. As the reader can easily see, this results in an exponential algorithm.

Our method is quite different; instead of using "brute force," we easily compute these $a_1, a_2, \ldots$ as the "positive lower root bounds" of polynomials, and thus, we are able to prove that, for an integral, univariate, square-free polynomial P, our method is $O(n^5L(|P|_\infty)^3)$.

This is the best theoretical computing time achieved thus far and empirical results verify the superiority of Akritas' method over all others existing.

Combinatorial Perspectives in Biostatistics and Epidemiology. *R. Grimson.* An introduction to some current thinking on the combinatorial nature of epidemiologic processes was given. Also, the practicality of combinatorics as an investigative tool was explored.

Analytic Functions and Conjugate Harmonic Functions. *R. Struble.* Analytic and conjugate harmonic functions were obtained directly from harmonic functions without resorting to integrations. Power series provided the most elementary technique, but any analytic continuation procedure will do.

Intersection Theory for Graphs and Projective Geometries. *T. Brylawski.* The concept of a matroid (an abstract coordinate-free analog of linear dependence) is defined. The way in which matroids may sometimes be represented by a graph or subset of a finite projective space was then used to settle some problems in graph theory, coding theory, and classical projective geometry.