Transmission of Correlated Gaussian Sources with Opportunistic Interference

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***Abstract*—Interfering transmissions of independent sources is shown to provide benefits at a cost of complex receiver design. In this paper, we study the optimum transmission of correlated Gaussian sources by allowing opportunistic interference in order to minimize the expected distortion. We assume a successive interference cancellation decoder at the destination and consider a Wyner-Ziv type setup to transmit the correlated sources along with interference. We show how interference can be utilized to improve the end-to-end distortion for correlated sources.**

I. Introduction

Sources in a wireless environment can be correlated as in Wireless Sensor Networks (WSN). In such networks in order to reduce the communication requirements, a commonly used technique is Distributed Source Coding (DSC) for which highly correlated sources are compressed separately at each terminal and decoded jointly. From a theoretical point of view, DSC finds its foundations in Slepian-Wolf and Wyner-Ziv compression schemes [1], [2]. Distributed source coding when individual descriptions can fail to reach the destination is studied for lossless compression in [3] and for distributed estimation (CEO problem) in [4]. In practice, different routing methods that exploit the correlation of the sources, have been proposed in WSN [5]- [6]. How the spatial correlation can be exploited on the Medium Access Control (MAC) layer is considered in [7]. By exploring data correlation and employing in-network processing, redundancy among sensed data can be curtailed and hence the network load can be reduced [8].

One characteristic of the aforementioned schemes is that their focus is on exploiting data correlation with source coding. The processed data from every source in the wireless network are transmitted with the help of a multiple access scheme that ensures orthogonal access to each specific source/transmitter. This approach is employed in order to simplify the design of the physical layer (PHY) receiver so that the demodulation algorithm requires only linear processing. Even though orthogonal transmissions are a natural choice for wireless transmission in a multiple access channel, it is not the only option.

One initially counter-intuitive idea is to allow the two sources to transmit simultaneously and allow their transmissions to interfere. However, the optimality of such a choice is generally an open problem. From an information-theory perspective, the capacity of this two-user Gaussian interference channel has been open for 30 years. The best known achievable region is that studied by Han-Kobayashi [9] but its characterization is very complicated. Recently, [10] provided a natural generalization of the classical notion of degrees of freedom for the point-to-point channel for interference-limited scenarios. The authors show that a very simple and explicit Han-Kobayashi type of scheme can achieve to within one bit the capacity of the two-user Gaussian interference channel. Furthermore, Katti et. al. [11] considered interference in a practical network and showed that interfering signals can be exploited to increase network capacity and that such an approach is implementable. The interplay between interference and data correlation for lossless data is first studied in [12]. In [13] an achievable rate distortion region for transmission of two correlated sources over a discrete memoryless interference channel (DMIC) as well as Gaussian interference channel is derived. Minero et. al. proposed an analog-digital scheme that achieves the best know performance for lossy communication of sources over the two-user discrete memoryless interference channel [14]. However, in these works, the authors followed an information theoretical approach to show the theoretical bounds and did not consider any practical protocols that exploits DSC along with opportunistic interference.

In this paper we study the optimum transmission of correlated Gaussian sources by allowing opportunistic interference in order to minimize the expected distortion. We use a Wyner-Ziv type setup which considers the compressed version of one of the sources as a remote side information for the compression of the other source. We assume a more complex interference canceling decoder at the receiver, so that we can allow two sources to transmit simultaneously. Then, a successive interference cancellation decoder is used for exploiting data correlation in order to recover the data symbols. We show

[1]Antonios Argyriou would like to acknowledge the support from the European Commission through the Marie Curie Fellowship WINIE-273041.

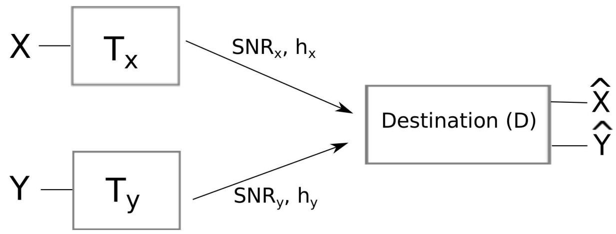


Fig. 1. Two terminal interfering transmission with correlated sources

that the estimation accuracy of the transmitted data samples can be improved and hence the distortion is minimized if we allow the transmitted signals to interfere. We also discuss how to achieve the optimal transmission strategy by allocating the channel to have opportunistic interference, i.e. controlling simultaneous/interfering transmissions from the two sources.

This paper is organized as follows. We introduce the system model in Section II. We review the effect of link losses on distributed compression in Section III and formulate the expected end-to-end distortion in Section IV. Section V analyzes the proposed strategies for different source and channel conditions. We conclude the paper in Section VI.

II. System model

A. Setup

Consider a system where T_x and T_y are two terminals in a wireless network communicating with a common destination. Each link has flat Rayleigh fading with instantaneous fading levels h_x and h_y, and average received signal to noise ratios SNR_x and SNR_y. The fading levels are accurately measured at the receivers, while the transmitters are only aware of the statistics. We define a channel frame as a block of N channel uses and assume the fading is constant for multiple channel frames.

We assume terminals T_x and T_y have access to two correlated sources X and Y respectively, which they wish to transmit to the destination with minimal expected distortion in squared error sense. The sources are zero-mean jointly Gaussian with the covariance matrix

$$K_{XY} = \begin{bmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{bmatrix} \tag{1}$$

where ρ is the correlation coefficient.

For the transmission of the correlated sources, we consider two different options. First, we assume time division multiple access (TDMA) among the terminals where each time slot lasts one channel frame. At each time slot, we send K source samples, leading to a bandwidth ratio $b = \frac{N}{K}$. For T_x, we assume the number of transmitted information bits (or source bits) per channel use is R_x. This results in a compression rate of $\overline{R}_x = \frac{NR_x}{K} = bR_x$ bits per source sample. Second, we let T_x and T_y transmit at the same time, hence, allowing interfering transmission. The terminals transmit at the optimum interference level by adjusting β which indicates the percentage of interference/overlap between two transmissions. In the maximum interference case, both terminals utilize the total transmission time slot and assuming T_x and T_y are each utilizing N channel uses with the TDMA, the total amount of channel uses for both terminals for the interfering transmission case becomes $2N$ where the compression rates remains the same. The minimum interference case is equivalent to TDMA transmission. Similar quantities can be defined for T_y as well.

B. Transmission Schemes

In order to illustrate the effects of the distributed compression and interference on correlated sources, we consider four transmission schemes. Below are the different modes which are summarized in Table I. The table shows the compression rate (in bits/source sample) and the number of channel uses for each terminal. In Section IV, we will optimize over these parameters to minimize the end-to-end distortion.

- $Mode\ A$: Each terminal compresses and transmits its own source directly to the destination in its own timeslot. The terminals ignore the source correlation.
- $Mode\ B$: This mode utilizes only distributed compression. We study a specific scheme where Y is compressed independently according to the rate distortion bound and X is compressed based on Y, but in a robust fashion realizing that the compressed version of Y may not be available at the destination. This is denoted as $T_x : (bR_x|Y)$ in Table I. The details will be discussed in Section III.
- $Mode\ C$: Each terminal compresses its own source without considering the correlation as in $Mode\ A$. However, in this mode interfering transmissions are allowed. The transmission of the terminals overlap for βN channel uses. The details will be discussed in Section IV.
- $Mode\ D$: In this mode, distributed compression along with interference is considered. The terminals transmit the correlated sources allowing interference as in $Mode\ C$ and by employing distributed compression as in $Mode\ B$.

III. Rate Distortion When Lossy Side Information May Be Absent

Before computing the expected distortions corresponding to the four transmission modes, in this section we first review the prior work on rate-distortion function for Gaussian sources when compressed side information may be absent. Finding a general rate-distortion region for distributed compression when the compressed streams may be lost is a difficult problem. That is why we concentrate on the asymmetric scenario where Y is compressed separately and X is compressed with respect to Y. Also for Gaussian sources, having side information at both the source encoder and the decoder does not improve the rate distortion performance over having it only at the decoder [2].

Without loss of generality, we can write, $Y = aX + Z$ when X and Y are jointly Gaussian with correlation matrix in (1). Here $Z \sim N(0, \sigma_z^2)$ is independent of X with $\sigma_z^2 = \sigma_y^2 - a^2\sigma_x^2$ and $a = \rho\frac{\sigma_y}{\sigma_x}$. Suppose Y is not available at X's encoder, and may or may not be available at the X's decoder. Let D_1 denote the squared error distortion achieved when Y is present

Modes	X's timeslot		Y's timeslot	
Mode A	$T_x : (bR_x, N)$		$T_y : (bR_y, N)$	
Mode B	$T_x : (bR_x \vert Y, N)$		$T_y : (bR_y, N)$	
Mode C	$T_x : (bR_x, \beta N)$ $T_y : (bR_y, \beta N)$	$T_x : (bR_x, (1-\beta)N)$	$T_x : (bR_x, \beta N)$ $T_y : (bR_y, \beta N)$	$T_y : (bR_y, (1-\beta)N)$
Mode D	$T_x : (bR_x \vert Y, \beta N)$ $T_y : (bR_y, \beta N)$	$T_x : (bR_x \vert Y, (1-\beta)N)$	$T_x : (bR_x \vert Y, \beta N)$ $T_y : (bR_y, \beta N)$	$T_y : (bR_y, (1-\beta)N)$

TABLE I
FOUR TRANSMISSION MODES, FOR EACH MODE THE COMPRESSION RATES AND THE NUMBER OF CHANNEL USES FOR EACH TERMINAL ARE SHOWN

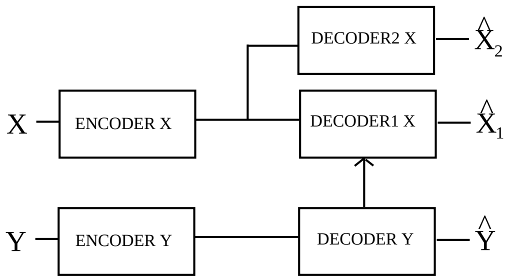


Fig. 2. Compression when lossy side information may be absent

at the destination, D_2 denote the distortion achieved when Y is absent.

In the distributed compression setup with unreliable links in Figure 2, Y is also compressed, hence, lossy. The rate distortion function of X when lossy side information may be absent, can be expressed as [15]:

$$R(D_1, D_2) = \begin{cases} \frac{1}{2}\ln\left(\frac{\sigma_x^2(\sigma_z^2+\sigma_w^2)}{D_1(a^2 D_2+\sigma_z^2+\sigma_w^2)}\right) & \text{if} \quad D_1 \le \sigma_2^2, D_2 \le \sigma_x^2 \\ \frac{1}{2}\ln\left(\frac{\sigma_x^2}{D_2}\right) & \text{if} \quad D_1 \ge \sigma_2^2, D_2 \le \sigma_x^2 \\ \frac{1}{2}\ln\left(\frac{\sigma_x^2(\sigma_z^2+\sigma_w^2)}{D_1(a^2\sigma_x^2+\sigma_z^2+\sigma_w^2)}\right) & \text{if} \quad D_1 \le \sigma_2^2, D_2 > \sigma_x^2 \\ 0 & \text{if} \quad D_1 \ge \sigma_2^2, D_2 > \sigma_x^2 \end{cases} \quad (2)$$

where $\sigma_2^2 = \frac{D_2(\sigma_z^2+\sigma_w^2)}{a^2 D_2+\sigma_z^2+\sigma_w^2}$, $\sigma_w^2 = \frac{\sigma_y^2 D_y}{\sigma_y^2 - D_y}$.

We are mainly interested in the compression rate of X in the regime $D_1 \le \sigma_2^2$, $D_2 \le \sigma_x^2$. In this region, for a given R_x, D_2 and D_y, the distortion D_1 is equal to

$$D_1(\overline{R}_x, D_2, D_y) = \frac{\sigma_x^2(\sigma_z^2\sigma_y^2+a^2\sigma_x^2 D_y)}{a^2 D_2\sigma_y^2 - a^2 D_2 D_y+\sigma_z^2\sigma_y^2+a^2\sigma_x^2 D_y}\, 2^{-2\overline{R}_x}$$

where $\overline{R}_x$ is the compression rate of X in bits per sample.

Finally, the side information Y is compressed using $\overline{R}_y$ bits per source sample and is sent directly to the destination with a distortion of:

$$D_y(\overline{R}_y) = \sigma_y^2 2^{-2\overline{R}_y} \quad (3)$$

IV. EXPECTED END-TO-END DISTORTION CALCULATION

For a given communication mode, average channel SNRs, source correlation and bandwidth ratio, the expected distortion is a function of the source rates and the amount of channel coding. We will assume that a complete frame will be discarded if the channel decoder can not correct all the errors.

Assuming that we have channel codes operating at rates R_x, R_y bits per channel use with corresponding compression rates $\overline{R}_x$=bR_x and $\overline{R}_y$=bR_y bits per source sample and using our formulation in Sec III, the average distortions for the most general strategy, $Mode\ B$, can be expressed in terms of error/success probabilities as

$$\begin{aligned} ED_x \quad = \quad & P^1\ D_1(bR_x, D_2, D_y) \\ & + P^2\ D_2 \\ & + P^3\ D_1(0, D_2, D_y) \\ & + P^4\ \sigma_x^2 \end{aligned} \quad (4)$$

$$\begin{aligned} ED_y \quad = \quad & (P^1+P^3)D_y(bR_y) \\ & + (P^2+P^4)\sigma_y^2 \end{aligned} \quad (5)$$

In the above formulation, D_y is given by (3). Also D_2 is the target distortion for X if the description of Y is lost. Furthermore the probabilities P^i are defined as the average probability of state (i) , where i denote whether (X, Y) is received. Here $i \in \{1, 2, 3, 4\}$. For $i = 1$, the compressed bits of both T_x and T_y are received, for $i = 2$ the compressed bits of T_x are received but the bits for T_y are lost, $i = 3$ means the compressed bits of T_y are received but the bits for T_x are lost and finally for $i = 4$ the compressed bits of both T_x and T_y fail to reach the destination.

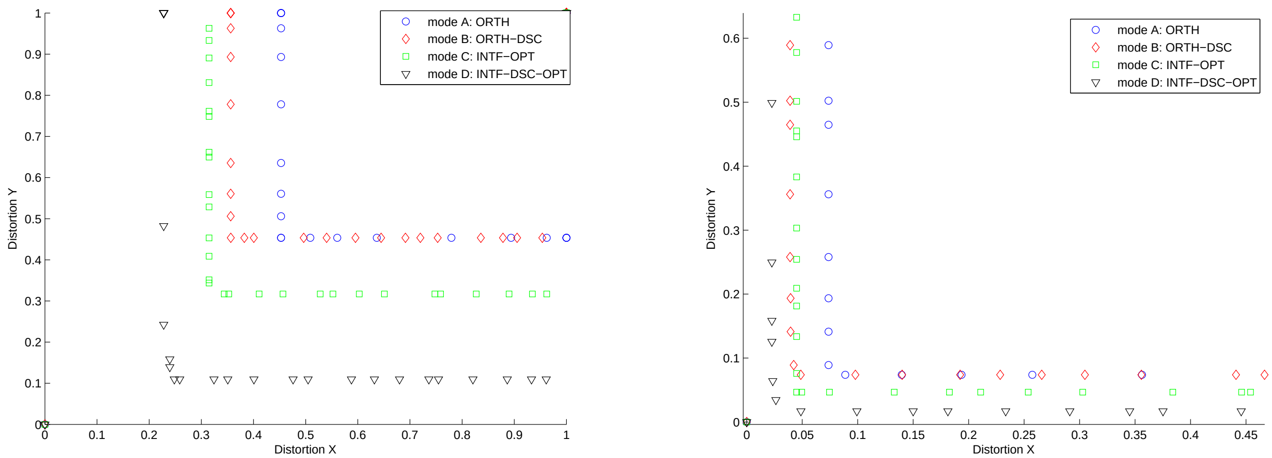
Next, we will illustrate the computation of the outage probabilities for TDMA based transmission as well as interfering transmission.

A. Outage Probability for TDMA based transmission

In order to compute the average probabilities P^i, we consider an information theoretic approach. Considering complex Gaussian codebooks, for a channel code operating at a rate R bits per channel use, information is lost when the instantaneous channel capacity is lower than R, leading to the outage probability $P_{out} = Pr\{C(|h|^2 SNR) < R\}$ for a point to point link where $C(x) = \log(1 + x)$ is the Gaussian channel capacity and $|h|$ is the fading amplitude.

We will illustrate the computation of P^1 as an example. Using the outage approach the compressed bits of both T_x and T_y are correctly received at the destination if

$$\begin{aligned} R_x &< C(|h_x|^2 SNR_x), \\ R_y &< C(|h_y|^2 SNR_y) \end{aligned} \quad (6)$$



(a) ED_x versus ED_y for SNR_x = SNR_y = 5dB (b) ED_x versus ED_y for SNR_x = SNR_y = 20dB

Fig. 3. ED_x versus ED_y for $\rho = 0.9$

Note that we have modeled the transmission of additional parity bits as independent Gaussian codebooks. Combining, we have

$$P^1 = Pr\{R_x < C(|h_x|^2 SNR_x), \quad R_y < C(|h_y|^2 SNR_y)\}$$

We can compute other probabilities P^i similarly and minimize (4) and (5) over all choices of R_x, R_y and D_2. This expected distortion minimization problem will be numerically carried out in Section V.

We note that for $Mode\ A$, we do not use the correlation at T_x, leading to

$$ED_x = (P^1 + P^2) D_x(bR_x) + (P^3 + P^4)\sigma_x^2 \quad (7)$$

B. Outage Probability for Interfering Transmissions and SIC

While computing the outage probability for the interfering transmissions, we follow a similar approach as in the TDMA case where we modify the SNR expressions such that interfering transmissions are accounted for. The signal model that is used for the case of interference is:

$$I = h_x X + h_y Y + W$$

Note that in the above expression X and Y are digital compressed signals. We assume an ordered SIC (OSIC) decoder is used which means that the stronger signal is decoded first. For exposition purposes, let us assume that the stronger signal is X. Then, the Signal to Interference plus Noise Ratio (SINR) for X can be expressed as:

$$SINR_x = \frac{|h_x|^2 \sigma_x^2}{|h_y|^2 \sigma_y^2 + N_0} \quad (8)$$

Now, after decoding X, we can decode Y with its respective SINR being equal to:

$$SINR_y = \frac{|h_y|^2 \sigma_y^2}{N_0} \quad (9)$$

The channel capacity under interference can be computed by replacing the $h_x SNR_x$ and $h_y SNR_y$ in (6) with the $SINR_x$, and $SINR_x$ respectively as computed above. We can now write the probability of receiving both X and Y for the case of interfering transmissions as,

$$P^1 = Pr\{R_x < 2\beta C(SINR_x) + (1-\beta)C(|h_x|^2 SNR_x), \quad R_y < 2\beta C(SINR_y) + (1-\beta)C(|h_y|^2 SNR_y)\},$$

We can compute other probabilities P^i similarly and minimize (4) and (5) over all choices of R_x, R_y and D_2. This expected distortion minimization problem will be numerically carried out in Section V.

We note that for $Mode\ C$, we do not use the correlation at T_x, hence the expected distortion of X is as in (7).

V. Results

In this section, we carry out the minimization of (4) and (5) numerically and compare the expected distortions achieved by different modes for various source correlation and channel link qualities. We assume $\sigma_x^2 = \sigma_y^2 = 1$ and $b = 1$. We consider a symmetric scenario where the terminals are at an equal distance to the destination such that $SNR_x = SNR_y = SNR$.

In order to observe the individual and joint effects of interference and distributed compression, for a fixed SNR and correlation coefficient, we vary R_x, R_y, β and D_2, compute the corresponding (ED_x,ED_y) pairs and plot the minimum values. Figure 3 illustrates the ED_x versus ED_y behavior for a high correlation coefficient ($\rho = 0.9$), for two different channel signal to noise ratios, SNR=5dB and SNR=20dB. Comparison of $Mode\ A$ and $Mode\ B$ shows how

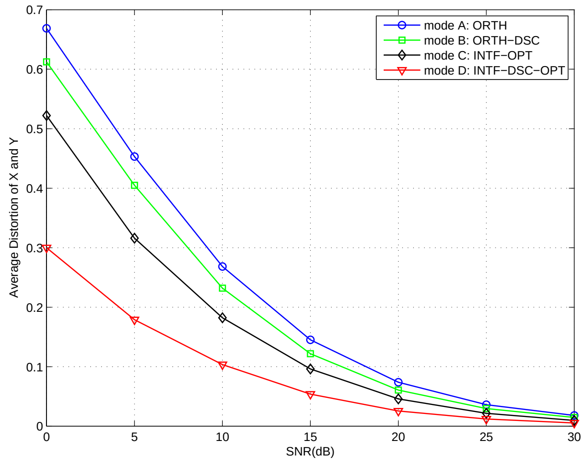


Fig. 4. Average distortion with $\rho = 0.9$, SNR_x = SNR_y = SNR

correlation helps to improve the distortion of X. Comparing $Mode\ C$ and $Mode\ A$, we illustrate the benefits of interfering transmission. Here note that allowing interfering transmission provides lower distortion Y, hence, reducing the distortion of X as well. Finally, $Mode\ D$ combines both the benefits of interference and DSC.

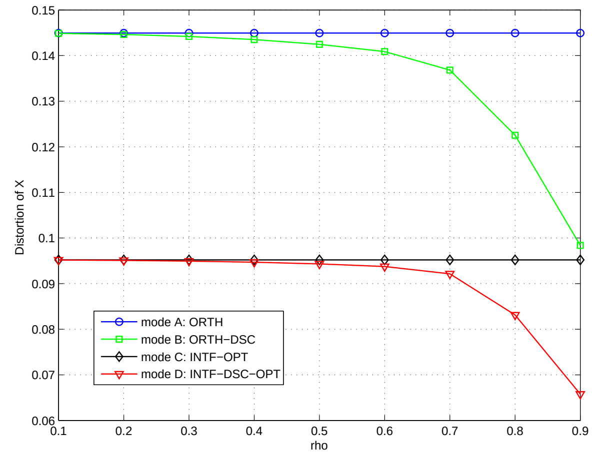
Note that a chosen rate pair jointly affects both ED_x and ED_y when distributed compression is used (Mode B and Mode D). One way to determine an optimal assignment of rates and the distortion trade off is to minimize the average distortion of X and Y, $\frac{ED_x+ED_y}{2}$. Figure 4 illustrates the achievable minimal average distortion with such optimal bit allocation over a wide range of channel SNRs for a high correlation coefficient ($\rho = 0.9$). We observe that distributed source coding along with opportunistic interference provides significant reduction to the end-to-end distortion.

Figure 5 illustrates the minimum distortion of X and Y as a function of correlation coefficient ρ. For $Mode\ B$, we observe the expected distortion of X reduces as the correlation increases. Both the distortions of X and Y are reduced in $Mode\ C$ independent from the correlation. For $Mode\ D$ we observe that, due to joint opportunistic interference and correlation, both the distortion of X and Y reduce as the correlation increases.

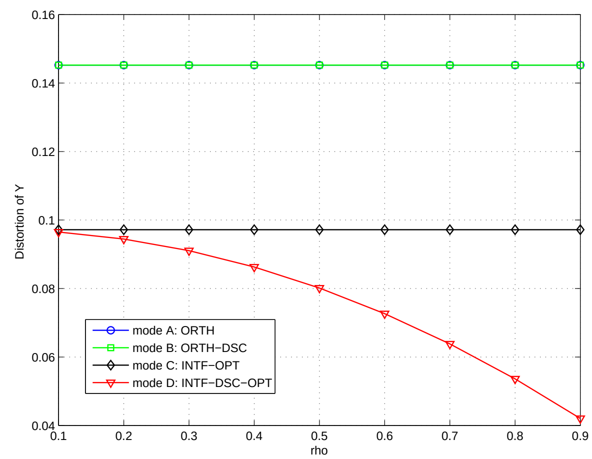
VI. Conclusion

In this paper we proposed compression and transmission strategies that make use of source correlation and interference. We showed that by allowing opportunistic interference, the expected distortion of correlated Gaussian sources can be minimized.

We argue that our approach offers a viable alternative for the transmission of correlated data in wireless fading channels. The reason is that complexity is shifted to the destination that is responsible for interference cancellation. Furthermore other benefits are expected since the channel access overhead through a MAC protocol is minimized.



(a) D_x versus ρ



(b) D_y versus ρ

Fig. 5. Distortion of X and Y over SNR_x = SNR_y = SNR

This paper only considers digital transmission of compressed sources over interference channel. A future direction is to extend this work by considering analog transmission as well as hybrid analog/digital transmission. Another direction is to take the correlation model into account at the decoder together with the interference and investigate the effectiveness of such a scheme compared to utilizing DSC at the source terminal.

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