Joint Estimation of the Number of Antennas and AoA of a Wireless Communication Transmitter

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***Abstract*---Estimating various parameters of a wirelessly transmitted information-bearing signal, besides the information itself, is essential for a number of applications (e.g. military fingerprinting, localization, radio-astronomy, etc.). In this paper we are interested in learning the number of antennas from which a wireless signal originates, and in addition its angle-of-arrival (AoA). Recent results from random matrix theory (RMT) allowed the development of highly efficient methods for detecting the number of antennas or the number of AoAs. In this paper we present an algorithm that applies RMT in two steps so that the receiver can detect simultaneously the number of antennas and the number of AoAs when a transmitter (Tx) with multiple antennas is present in an environment with several scatterers. Our simulations reveal an algorithm with very low estimation errors for a randomized placement of scatterers.**

***Index Terms*---Uniform linear array, number of antennas, AoA estimation, Akaike Information Criterio (AIC), random matrix theory.**

I. Introduction

A wireless communication radio frequency (RF) signal contains the so-called *side information*, that is information that reveals parameters of the transmitter and the user that are independent of the the data. Some examples include the number of antennas from which the wireless signal was transmitted and the angle-of-arrival (AoA) of the RF signal at a particular receiver (Rx). In this paper we deal with the problem of estimating these two specific parameters when the transmitted signal experiences multipath scattering and fading between itself and the receiver (Fig. 1). To solve this dual-objective problem we start by employing the most sophisticated up-to-date methods for each of these two problems, namely random matrix theory (RMT) [1] for number of antennas estimation, and for AoA estimation the super-resolution multiple signal classification (MUSIC) algorithm [2] .

As expected, these two problems have been thoroughly explored in the literature but always under specific settings. The problem of estimating how many independent sources does a signal contain under Gaussian noise was studied in [3]. That algorithm calculates the covariance matrix of the aggregate signal, then its eigenvalues, and finally uses the Akaike Information Criterio (AIC) for estimating the number of sources through model selection. This has emerged as a classic *non-parametric* technique for this problem. The same AIC metric (or similar ones) can be used for calculating how many antennas a wireless transmitter uses in Multiple-Input Multiple Output (MIMO) wireless systems when each antenna transmits data from an independent source [4]. More recent works improve upon non-parametric estimation by using results from random matrix theory (RMT) [1]. Detecting the number of sources in a signal can also be accomplished with *parametric* methods that have better performance. However, these methods assume knowledge of the signal model which means knowledge of the covariance matrix form [5], unlike [3]. These methods of course fail when the number of sources differs from the number of antennas. Furthermore, in wireless fading channels the signal model, and so the covariance matrix of the receiver baseband signal, depends on the unknown channel realization making thus the channel coefficients a nuisance parameter. Hence, non-parametric methods are preferable.

Regarding estimation of the AoA there is a plethora of techniques all of which require an array of antennas at the receiver, as the topology in Fig. 1 illustrates. For achieving high angular resolution *subspace methods* focus on the covariance matrix structure of the received signal. We explore the most widely used method namely MUSIC [2].

In our recent work [6] we have shown that these two problems are coupled: The first requirement for estimating the AoA with the MUSIC algorithm is the need for knowing the precise number of angles from which the scattered signals arrive. What we have observed is that since in a multipath channel the signals over the multiple paths are correlated, AIC/RMT-based methods are robust to correlation and they can detect the antenna number. However, the effects of multipath must be removed from the signal before we use the same methods for estimating the number of AoAs. This now allows the solution of both problems sequentially [6]. In this paper, we propose an algorithm that is based first on RMT for sequential antenna/AoA estimation, and finally on MUSIC for AoA estimation.

II. System Model

The communication system model is illustrated in Fig. 1 and is based on a multi-antenna transmitter, an unknown number of scatterers (just one is illustrated in Fig. 1), and an unauthorized receiver (URx) named Eve that uses a uniform linear arrays (ULA) of antennas. The wireless modulated signal is narrowband. The goal of Eve is to find the number of transmitter antennas and the AoA of the incident signal. Therefore, our discussion concerns the URx.

Signal Model: The ULA consists of N_{Rx} elements that are separated by d meters. One typical assumption is that at the URx the impinging waves at the ULA are approximated as specular plane waves that arrive in parallel at the Rx. This

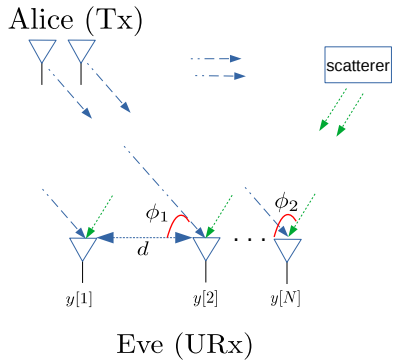

Fig. 1: System model with a multi-antenna transmitter and the unauthorized receiver with a ULA.

holds when the distance between the Tx/Rx is significant. A static Rayleigh flat fading channel is also assumed and the complex fading coefficient of the baseband model is denoted with h. If we account for the AoA ϕ and AoD θ of the signal as a single phase term $\eta(\phi,\theta)$ then for a path between the Tx and the URx the overall complex baseband channel gain is $h\exp(j\eta(\phi,\theta))$. Without loosing generality in our analysis we consider only the AoA. So in our model we have separated the two contributing components in the baseband channel gain first into the Rayleigh complex gain and second in the steering vector of the ULA as we describe next in more detail. Let us first model the modulated signal of the transmitter as an $N_{\text{Tx}} \times 1$ vector $\mathbf{s}$. The data across the antenna elements are assumed to be uncorrelated and are contained in $\mathbf{s}$. To capture the copies of this signal across the M paths this signal is repeated M times and packed in the $MN_{\text{Tx}} \times 1$ vector $\mathbf{x} = [\mathbf{s}^T ... \mathbf{s}^T]^T$. Note that the covariance matrix $\mathbf{C_x}$=$\mathbb{E}[\mathbf{x}\mathbf{x}^H]$ has dimensions $MN_{\text{Tx}}\times MN_{\text{Tx}}$ and rank N_{Tx}, i.e. it is not full rank. We are now ready to express the signal model at the ULA receiver as

$$\mathbf{y} = \mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{x} + \mathbf{w}, \tag{1}$$

where $\mathbf{H}_{\text{iid}}$ ($MN_{\text{Tx}}\times MN_{\text{Tx}}$) is a block diagonal matrix that models the iid baseband channel samples as a result of Rayleigh fading:

$$\mathbf{H}_{\text{iid}} = \text{diag}(\underbrace{h_1 \ ... \ h_1}_{N_{\text{Tx}} \text{ copies}} \ ... \ h_M \ ... \ h_M)$$

Of course $\mathbf{w}$ is the vector of AWGN samples that has power σ^2. The ULA steering matrix is $\mathbf{A}$ with dimensions $N_{\text{Rx}}\times MN_{\text{Tx}}$. The columns of $\mathbf{A}$ contain the steering vectors that model phase differences between the receiver signal and the ULA antenna elements. The i-th column corresponds to the i-th AoA:

$$\mathbf{a}^T(\phi_i) = [1 \quad e^{j2\pi f_c \frac{d\cos\phi_i}{c}} \quad ... \quad e^{j2\pi f_c \frac{(N_{\text{Rx}}-1)d\cos\phi_i}{c}}] \tag{2}$$

The carrier frequency is f_c and the additional time that is required for the signal to travel between two neighboring array elements is $d\cos(\phi_i)/c$ and is easily deduced with trigonometry from Fig. 1. Consequently, with M AoAs this $N_{\text{Rx}}\times MN_{\text{Tx}}$ matrix is:

$$\mathbf{A} = \begin{bmatrix} 1 & .. & 1 \\ ... & ... & ... \\ \underbrace{e^{j2\pi f_c \frac{(N_{\text{Rx}}-1)d\cos\phi_1}{c}}}_{N_{\text{Tx}} \text{ copies}} & .. & e^{j2\pi f_c \frac{(N_{\text{Rx}}-1)d\cos\phi_M}{c}} \end{bmatrix} \tag{3}$$

It is also clear that as more antennas are added at the i-th transmitter the corresponding i-th column is replicated in the matrix.

III. Antennas and AoA Estimation

Now we proceed to discuss how the number of antennas N_{Tx} and the M multipath AoAs are estimated. The covariance matrix is calculated from the data with the classic unbiased estimator $\widehat{\mathbf{C}}_{\mathbf{y}} = \frac{1}{\#\text{samples}}\mathbf{y}\mathbf{y}^H$.

A. AIC/MDT Metrics

Non-parametric methods estimate how many independent signals exist in the received signal using the covariance matrix, and more specifically its eigenvalues. In our case the receiver has only an estimate of $\mathbf{C}_{\mathbf{y}}$, namely $\widehat{\mathbf{C}}_{\mathbf{y}}$. This may be an issue since a subset of the eigenvalues correspond to the noise subspace. In the first work that studied this problem [3], the eigenvalues of $\widehat{\mathbf{C}}_{\mathbf{y}}$ constitute the proposed metric in a channel experiencing only AWGN. Now let l_i denote the i-th eigenvalue of $\widehat{\mathbf{C}}_{\mathbf{y}}$. Then the AIC is:

$$\text{AIC}(m) = -2(N_{\text{Rx}} - m)T\log\left(\frac{\prod_{i=m+1}^{N_{\text{Rx}}} l_i^{1/(N_{\text{Rx}}-m)}}{\frac{1}{N_{\text{Rx}}-m}\sum_{i=m+1}^{N_{\text{Rx}}} l_i}\right) + 2m(2N_{\text{Rx}} - m) \tag{4}$$

The transmit antennas that are estimated is equal to the value m that minimizes the AIC metric, i.e.

$$\widehat{N}_{\text{Tx}} = \arg\min_{m=0,..,N_{\text{Rx}}} \text{AIC}(m) \tag{5}$$

The minimum distance length (MDL) metric can be calculated similarly [3] and is not presented due to lack of space.

However, the randomness of $\widehat{\mathbf{C}}_{\mathbf{y}}$, and that of its eigenvalues allows us to use tools from RMT for improving upon estimation. The RMT estimator reported in [1] is also adopted here. The number of antennas or AoAs can be estimated as:

$$\widehat{N}_{\text{Tx}} = \arg\min_{i=0,..,N_{\text{Rx}}} \{l_i < \sigma^2 C_{\text{th}}(\mu,\xi,\alpha)\} - 1 \tag{6}$$

The threshold constant C_{th} depends on the centering μ and scaling ξ parameters of the Tracy-Widom distribution and α is a desired false alarm rate [1].

In this paper the AIC/MDT metrics are used for estimating jointly the number of antennas and AoAs as we describe next.

B. Sequential Estimation Method

To clarify the rationale of our algorithm consider the simplest case with zero spatial correlation in the ULA antennas elements and so (1) is simplified leading to the MIMO i.i.d. channel:

$$\mathbf{y} = \mathbf{H}_{\text{iid}}\mathbf{x} + \mathbf{w} \tag{7}$$

For the matrix $\mathbf{H}_{\text{iid}}$ note that its rank is rank($\mathbf{H}_{\text{iid}}$)=min$(N_{\text{Rx}}, MN_{\text{Tx}})$ since $\mathbf{H}_{\text{iid}}$ has N_{Rx} rows. The covariance matrix of the complete signal $\mathbf{C_s}$=$\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^H$ is a $N_{\text{Rx}}\times N_{\text{Rx}}$ matrix. So rank($\mathbf{C_s}$)=min$(N_{\text{Rx}}, N_{\text{Tx}})$ and in case $N_{\text{Rx}}\geq N_{\text{Tx}}$ the rank is equal to the number of transmitter antennas [4].

However, for our model in (1) the $N_{\text{Rx}}\times N_{\text{Rx}}$ *signal* covariance matrix is $\mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^H\mathbf{A}^H$. Recall also that for our model in (1) $\mathbf{H}_{\text{iid}}$ is a diagonal matrix that has dimensions $MN_{\text{Tx}}\times MN_{\text{Tx}}$ and is full rank. So it will be rank($\mathbf{C_s}$)= min$(N_{\text{Rx}}, N_{\text{Tx}}, M)$. When $N_{\text{Rx}} \geq M > N_{\text{Tx}}$ the rank corresponds to the transmitter antennas. *What we propose is to estimate first the number of antennas N_{Tx}, then perform spatial smoothing so that $\mathbf{C_x}$ becomes full rank (equal to MN_{Tx}), and then estimate M from the smoothed matrix which now has rank*($\mathbf{C_s}$)= min(N_{Rx}, MN_{Tx}, M). In Table I where we present the usefulness of our algorithm we see that our idea cannot estimate N_{Tx} when the number of used antennas is larger than the number of paths.

Antennas per User	$<$ Paths	$\geq$Paths
Algorithm result	AoAs,Antennas	AoAs

TABLE I: Suitability of the proposed algorithm for number of antennas and AoA estimation.

C. Spatial Smoothing

The final critical part of the algorithm is the use of spatial smoothing [7]. Spatial smoothing restores the rank of $\mathbf{C_s}$ by grouping the ULA into L subarrays. The smoothed estimate for a number of L subarrays is:

$$\bar{\mathbf{C}}_{\mathbf{y}}^{(L)} = \mathbf{A}\mathbf{H}_{\text{iid}}\bar{\mathbf{C}}_{\mathbf{x}}^{(L)}\mathbf{H}_{\text{iid}}^H\mathbf{A}^H + \sigma^2\mathbf{I} \tag{8}$$

This means that even if this algorithm can create a covariance matrix with the desired rank MN_{Tx}, the rank of $\mathbf{A}$ (which is M) is not affected. This observation is exploited by our overall algorithm (as we will soon explain with more details) that its core principle is to calculate after spatial smoothing AIC/RMT again so as to get the AoAs M.

IV. AoA Estimation with MUSIC

The receiver at the ULA calculates the AoA of the several incoming wireless signals that are linearly superposed, by using the phase difference of the signal at different antennas (as a result of different time of arrival (ToA)). The beamsteering vector $\mathbf{a}^T(\phi)$ in (2) expresses the difference in the phase of a signal as a result of this ToA difference. By applying the MUSIC algorithm we can then derive the AoA [2], [8]--[10]. MUSIC works only if the number of AoAs M is known, making the use of the algorithms in the last section a necessity. With MUSIC we use Eigenvalue Decomposition (EVD) for $\widehat{\mathbf{C}}_{\mathbf{y}}$. There are two types of eigenvectors that correspond to the signal and noise sub-spaces. There are M AoAs so if we pack the eigenvectors into matrices we have the first matrix $\mathbf{Q}_1$=$[\mathbf{q}_1, ..., \mathbf{q}_M]$, and the matrix for the eigenvectors that have eigenvalues with value 0 (zero-value eigenvectors) is $\mathbf{Q}_2$=$[\mathbf{q}_{M+1}, ..., \mathbf{q}_{N_{\text{Rx}}}]$. The main idea of MUSIC is that the noise sub-space is orthogonal to signal space, that is $\mathbf{a}^H(\phi)\mathbf{Q}_2$=0. Based on this we can define a function that allows us to calculate the AoAs and is called the MUSIC pseudo-spectrum:

$$P_{\text{MUSIC}}(\phi) = \frac{1}{\mathbf{a}^H(\phi)\mathbf{Q}_2^H\mathbf{Q}_2\mathbf{a}(\phi)} \tag{9}$$

It is easy to see that the peaks in $P_{\text{MUSIC}}(\phi)$ contain the AoAs.

Algorithm 1: High-level pseudo-algorithm for joint antennas estimation and AoA estimation

Input: $\mathbf{y}, N_{\text{Rx}}$
Output: $\widehat{M}, \widehat{N}_{\text{Tx}}, \hat{\boldsymbol{\theta}}$

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Estimate Ĉ_y;
Estimate # of ants. with RMT as N̂_tmp;
Calculate smoothing array size L = N_Rx − 2;
Calculate smoothed cov. matrix C_y^smooth with (8);
Estimate # of AoAs with AIC,MDL or RMT as M̂;
Estimate MUSIC AoAs θ̂ with M̂ sources;
if N̂_tmp < M̂ then
    N̂_Tx ← N̂_tmp
else
    Cannot decide on N_Tx;
end
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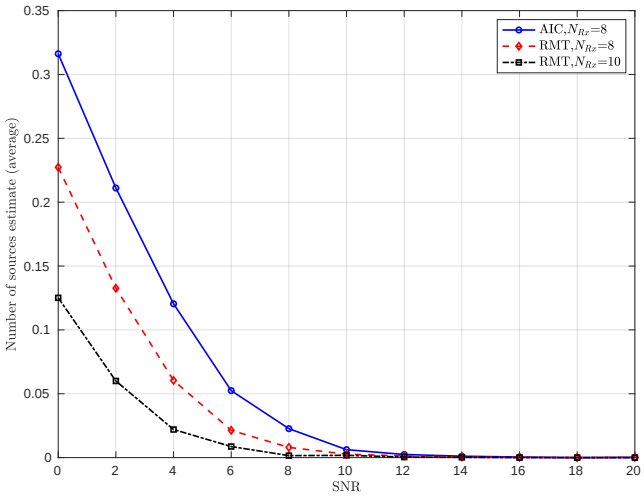
V. Algorithm

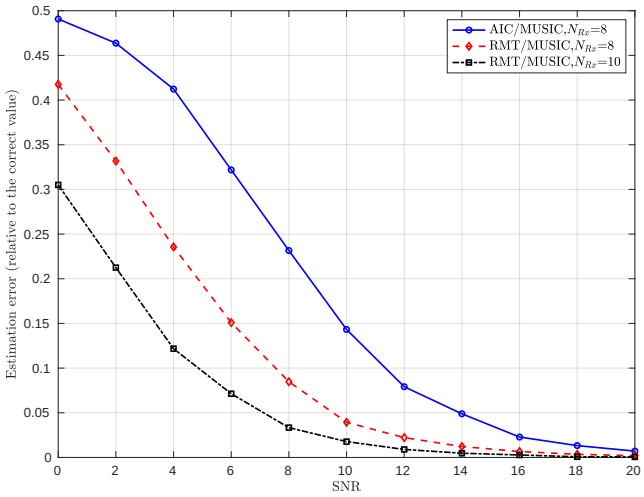
Our proposed algorithm is illustrated in Algorithm 1, and it effectively summarizes the analysis we did in the previous sections. The covariance matrix is estimated first from the data, and then the antennas are estimated with the desired AIC or MDT metric. This last estimate of N_{Tx} is temporary because we must also calculate the AoA number which will allow our algorithm to compare the two numbers and decide on the number of antennas (see the discussion in III-B). Spatial smoothing is the next step of the overall algorithm. The length of the sub-array L in the spatial smoothing algorithm has to be larger than the number of correlated signals (M in our case including LOS). However, at this point of the algorithm M is unknown and so we set L to be the maximum value that it can take namely $N_{\text{Rx}} - 2$. After deciding the sub-array size the algorithm executes sequentially the steps for estimating the smoothed covariance matrix, recalculates the AIC or MDT metrics for obtaining the estimate $\widehat{M}$, and also calculates the MUSIC pseudo spectrum that allows calculation of the AoA vector $\hat{\boldsymbol{\theta}}$. Finally, the algorithm decides on the number of antennas when the condition $\widehat{N}_{\text{tmp}} < \widehat{M}$ is valid.

VI. Simulations

The performance of the proposed algorithm is evaluated with a straightforward setup that focuses on fundamental system parameters. The ULA is critically configured, that is d=$\lambda/2$. We explored a different number N_{Rx} for the ULA



(a) N_{Tx} estimation error for the AIC and RMT source estimators and different number of ULA antennas.



(b) AoA estimation error with MUSIC for the AIC and RMT source estimators and different number of ULA antennas.

Fig. 2: Results for the estimation error.

antenna elements. The transmitted signal has B=1MHz, it is BPSK and it is modulated on a WiFi 5GHz carrier. We assume the availability of 10 data snapshots $\mathbf{y}$ that are used for estimating the covariance matrix. Since our goal is to derive accurate estimates of the number of antennas and AoAs when scatterers are at random locations, we tested for each ULA receiver SNR 100 different transmitter locations leading to random AoAs. The transmitter location is uniformly and randomly distributed. The y axis illustrates the estimation error that is relative to the correct value of the desired parameter for different receiver SNRs illustrated in the x axis. Furthermore, we configured the actual number of paths in the random topologies to be on average 1 more than the number of transmitter antennas in each topology e.g., 3 paths and 2 antennas, 5 paths and 4 antennas, etc.

Regarding the error for the number of antennas estimation it is illustrated in Fig. 2(a). As expected the RMT estimator offers superior performance for the same number of ULA antennas at the URx. More importantly the combination of more ULA antennas and the RMT estimator offer even better performance. But note that our actual interest is how all the parameters $\widehat{M}, \widehat{N}_{\text{Tx}}, \hat{\boldsymbol{\theta}}$ are estimated. Next, $\widehat{M}$ is estimated after spatial smoothing and finally MUSIC estimates $\hat{\boldsymbol{\theta}}$. Fig. 2(b) results demonstrate that when the SNR and the number of ULA antennas is increased this improves the AoA estimate with MUSIC. Overall we notice that the dual estimation problem offers better performance with higher SNR/N_{Rx} as desired and RMT is the estimator of choice for estimating the number of sources/antennas.

VII. Conclusions

Estimating jointly the number of transmitter antennas and AoAs of a wireless communication transmitter is a challenging problem even with RMT-based estimators. The reason is that the result must be interpreted depending on the precise number of correlated signals and the number of scatterers/paths. In this paper we focused on identifying multipath scenarios where these estimators can be safely applied in a specific order allowing us to solve the joint problem of number of antennas and AoA estimation. Our results showed the very good performance of the proposed two-step RMT-based estimation algorithm under different conditions.

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