Passive Angle-Doppler Profile Estimation for Narrowband Digitally Modulated Wireless Signals

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***Abstract*—In this paper, we improve estimation of the Doppler shift present in signals that originate from multiple wireless digital communication transmitters. We deploy a uniform linear array (ULA) that overhears the frequency band of interest, hence an unauthorized wireless receiver (URx), and propose an algorithm for extracting the symbol duration from the unknown modulated data. The baseband wireless modulated signal is stripped from its data-induced phase shifts, allowing us to calculate high quality estimates of the periodogram, the spectrogram, and the angle-Doppler profile. Performance results show that high quality results without artifacts from digital modulation can be obtained.**

I. Introduction

Wireless communication signals typically contain a wealth of information besides the digital data themselves. This additional information may concern the communication channel, or the transmitter (Tx) and its behavior. One specific parameter of the wireless signal, namely the Doppler shift that the signal experiences, can reveal the speed of the transmitter [1]. However, calculating the Doppler shift in a wireless signal that went through a channel is easier when we know the original signal (e.g. through preambles), while it is more challenging in passive/blind systems where there is no cooperation between the transmitter of the signal and an unauthorized Rx (URx). More precisely wireless digitally modulated signals contain amplitude/frequency/phase shifts that depend on the data and Tx/Rx imperfections and are not due to Doppler. Another problem for a blind URx that receives and processes a wireless signal over time is to distinguish the sources of the transmissions. For example assume that we have two static WiFi users and our blind URx desires to calculate the Doppler shift. For the URx the received baseband signal over time is effectively a sequence of I/Q data (with just noise data between frame transmissions) and is unaware of which set of data to feed to the algorithm that is responsible for Doppler estimation. Consequently, the problem at hand requires the passive/blind estimation of the Doppler shift of multiple wireless sources that use the same frequency band.

Doppler estimation in classic communications systems can be performed with the aid of known preambles either with maximum-likelihood (ML) estimation, model selection, or with the discrete Fourier transform (DFT) [2]. But these systems assume a known preamble at the receiver. In a passive systems where Doppler estimation is performed by a URx the content of the preamble is not known. In this class of so-called *passive RADAR systems* there are two approaches for estimation of the Doppler shift in a signal. First, systems that still

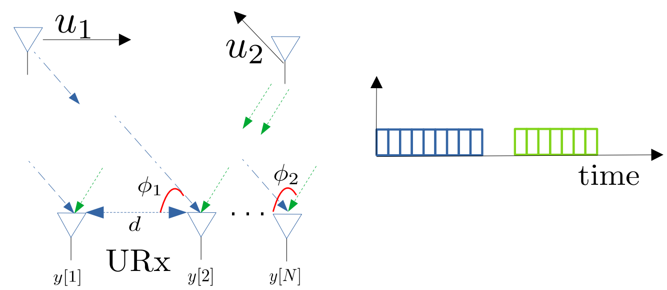


Fig. 1: Representative topology with two users/transmitters and the URx (left), and frame transmission from them (right).

use channel state information (CSI) extracted from a wireless protocol preamble (even without knowing the preamble), and systems that are completely agnostic to the wireless protocol and use RADAR techniques. Passive WiFi RADAR (PWR) systems of the literature are developed on the premise that there are two signals at the URx, typically arriving with a time offset because of multipath. These methods correlate directly the received signal from the Tx and the reflected signal from a moving target of interest [3]. However, since one of these signals contains Doppler and the other one does not, there is a need to cross-correlate the signals. When the URx has the signal from a single source without its reflections there is no reference to calculate Doppler. Hence, a different approach is needed when no additional sources of the signal exist.

In this paper, we focus on improving estimation of the Doppler shift present in received signals that originate from multiple wireless digital communication transmitters. We deploy a URx with a uniform linear array (ULA) that consists of N_{Rx} antennas. We propose an algorithm for extracting the symbol duration from an unknown signal. Second, the baseband wireless modulated signal is then stripped from its data-induced phase shifts, allowing us to calculate the periodogram and the Doppler shift. With the help of the ULA we also calculate the spectra for different AoAs as described above. *The AoA-Doppler profile we produce is the combination of these data and it eventually allows us to separate wireless sources through their AoA.* The advantages of the proposed scheme relative to related work are the following: 1) We create the AoA-Doppler profile but for a wireless network where users transmit over different time slots. 2) Unlike PWR systems ours does not suffer from interference of the original signal that does not experience Doppler. 3) There is no need to receive a second copy of the signal for performing correlation and calculating Doppler.

II. Preliminaries

A. Signal Model for one Transmitter and one URx Antenna

In a wireless environment where several users operate in a frequency band we are interested to calculate the Doppler frequency of the received signal of each user f_D. To limit the scope of the problem in this work we assume that the URx monitors a given frequency band (typical wireless receivers are narrowband). The signal in this band is down-converted assuming a center frequency of $f_c^{'}$ Hz at its center. During down-conversion there will be a mismatch between the actual carrier frequency f_c of passband signal and $f_c^{'}$ that is due to the carrier frequency offset (CFO) of f_{CFO} Hz between the local oscillators (LO) at the Tx and our URx and also because of Doppler of f_D Hz. These two phenomena can be effectively distinguished with ML estimators and algorithms that are independent of our scheme (e.g. see [2]), and so we do not include CFO in our model and simulation. As a result the baseband continuous time signal model for a narrowband flat fading channel with a Doppler frequency of f_D Hz is

$$y(t) = hxe^{j\pi 2 f_D t} + w,$$

where h is the complex channel gain excluding Doppler and CFO, x are the phase-modulated data, and w is the AWGN sample. In our model we can safely ignore the sampling clock offset (SCO) since we are not interested in the correct demodulation of a symbol (by sampling when the matched filter output peaks). Sampling with a rate of f_s Hz, we obtain the discrete model for flat slow-fading:

$$\begin{aligned} y[n] &= hx[n]e^{j\pi 2 f_D n/f_s} + w[n] \\ &= |hx[n]|e^{j\pi 2 f_D n/f_s + j\phi[n]} + w[n] \end{aligned} \tag{1}$$

At one antenna of the ULA receiver we collect a snapshot $\mathbf{y}$ of N samples in total from the model in (1), all experiencing the same h due to flat fading. The vector of the samples where $x[n]$ remains unchanged (and so all the samples correspond to the same symbol x) is denoted as $\mathbf{y}_i$, with the number of samples being N_i. Of course $\mathbf{y}_i$ and N_i are unknown since we do not know at the URx when symbol transitions take place.

B. Signal Model for the Multiple Sources and the ULA

For our complete signal model we pack the N transmitted samples $x[n]$ from a number of M wireless sources in the $M \times N$ matrix $\mathbf{X}$. Similarly we create the matrix for the channel $\mathbf{H}$ ($M \times M$), and the Doppler element in (1) that for each source i is $e^{j\pi 2 f_{D_i}/f_s}$ and is added in a diagonal matrix $\mathbf{F}$ ($M \times M$). The N samples from all the N_{Rx} antennas in the ULA are given by the $N_{\text{Rx}} \times N$ received signal matrix $\mathbf{Y}$:

$$\mathbf{Y} = \mathbf{AHFX} + \mathbf{W} \tag{2}$$

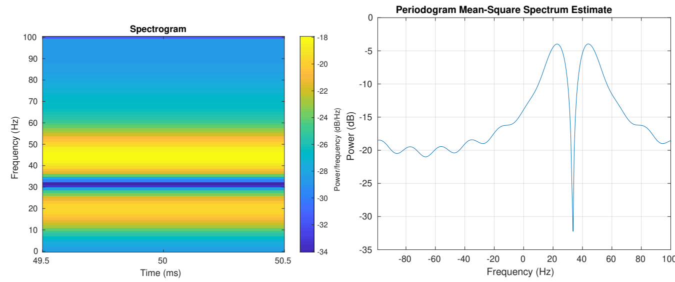
$\mathbf{A}$ is the unknown $N_{\text{Rx}} \times M$ steering matrix of the ULA. Each column of $\mathbf{A}$ contains the steering vector that captures the phase difference between the received signal at each element of the ULA that originates from the i-th AoA:

$$\mathbf{a}^T(\phi_i) = [1 \quad e^{j2\pi f_c \frac{d\cos\theta_i}{c}} \quad \ldots \quad e^{j2\pi f_c \frac{(N_{\text{Rx}}-1)d\cos\theta_M}{c}}] \tag{3}$$

In this model $d\cos(\theta_i)/c$ is the additional time required for the RF signal to travel between two antenna elements of the ULA (Fig. 1 illustrates the geometry). Consequently, if we assume we have M sources/AoAs this $N_{\text{Rx}} \times M$ matrix is:

$$\mathbf{A} = \begin{bmatrix} 1 & .. & 1 \\ \ldots & \ldots & \ldots \\ e^{j2\pi f_c \frac{(N_{\text{Rx}}-1)d\cos\theta_1}{c}} & .. & e^{j2\pi f_c \frac{(N_{\text{Rx}}-1)d\cos\theta_M}{c}} \end{bmatrix} \tag{4}$$

Now for this signal model $N, h, x[n], f_{D_i}, \theta_i$ are unknown.



(a) (b)

Fig. 2: Spectrogram (for a single snapshot), and periodogram before our algorithm is applied.

C. Challenges

Over the duration of the constant channel fade h data are collected as a single snapshot of $L = \frac{N}{f_s}$ seconds and are fed to the spectrum (periodogram or spectrogram) estimator. The spectrum is estimated for every snapshot. Fig. 2 illustrates the spectrogram that identifies two frequencies 22Hz and 44Hz, while the real Doppler is 33Hz and corresponds to a user moving at 10m/s and transmitting BPSK data. Clearly, this is not how we would like the spectrogram to look for a specific snapshot. But this is expected as we analyze next. We know that for a sampled rectangular pulse of N samples (starting from $n = 0$) and its frequency being at f_D Hz the discrete time Fourier transform (DTFT) is:

$$X(f) = \alpha \frac{\sin[\pi(f - \frac{f_D}{f_s})N]}{\sin[f - \frac{f_D}{f_s}]} e^{-j2\pi(f - \frac{f_D}{f_s})\frac{N-1}{2}}$$

In the above the aliased sinc function is multiplied with the exponential term, that when added, accentuates the spectral leakage lobes. Now consider the data signal in this paper and a subset $\mathbf{y}_1$ of the snapshot $\mathbf{y}$ where the symbol is positive, i.e. $x[n] = 1, \forall y[n] \in \mathbf{y}_1$. The DTFT (with no AWGN) is denoted as $Y_1(f)$, while for the next symbol, a second subset when the data symbol is negative the DTFT is denoted as $Y_2(f)$. Also $a = |hx|\exp^{j\phi}$, and $Y(f) = Y_1(f) + Y_2(f)$. To generalize our observations consider that within the snapshot there are a number of P symbols, and also assume that since several symbols are sampled that the duration of each symbol is sampled and contained in $\mathbf{y}$ (in terms of samples is $\frac{N}{P}$). The DTFT is

$$\begin{aligned} Y(f) &= \sum Y_i(f) \\ &= \frac{\sin[\pi(f-\frac{f_D}{f_s})\frac{N}{P}]}{\sin[f-\frac{f_D}{f_s}]} \sum_{p=1}^{P} \alpha_i e^{-j2\pi(f-\frac{f_D}{f_s})[(p-1)(\frac{N}{P}-1)+\frac{\frac{N}{P}-1}{2}]} \end{aligned} \tag{5}$$

Now we elaborate on the calculation of $Y(f)$. First, due to the symmetry across the zero frequency component of all $Y_i(f)$'s, their summation means that the same is true for $Y(f)$: that is a peak at a certain positive frequency is also present at the negative frequency. Now either adding or subtracting theses pulses (this happens because of the different α_i of each pulse) the symmetry across f_D is preserved. What we also notice is that $|Y(f)|$ has a null at $\frac{f_D}{f_s}$ when we have an equal number of $\pm$ pulses within the snapshot while it does not have this null when there are either more + or -, hence a non-zero value at f=0. Regarding the peaks we must note that it is well known that they are not trivial to derive even for the sinc function [4], and consequently for its shifted version. This means that we cannot analytically calculate the new peaks originating from the addition or subtraction of these pulses.

Hence, the problem with this type of modulated data is that 1) the location of the peak (or that of two symmetrically located peaks) in the spectra depends on the polarity and number of symbols (even/odd) we use for calculating the DFT, 2) the sidelobes have higher power, and are closer to the frequency f_D/f_s as we include more symbols into $\mathbf{y}$. 3) We cannot analytically calculate the peaks. The objective of our algorithm is to calculate f_D/f_s given the previous problems.

III. Algorithm

The baseband signal after the demodulator at the receiver is sampled at a rate of f_s Hz. For one antenna the snapshot $\mathbf{y}$ consists of N samples, while there are N_{Rx} antennas. The DFT of the snapshot is calculated and since it is in baseband the frequency peaks correspond to the Doppler frequency f_D. In the first part of the algorithm illustrated in Algorithm 1, we employ AoA beamforming first. Then, we execute the SDBA, while next we perform phase correction, and finally calculate the periodogram and the AoA-Doppler profile.

Algorithm 1: High-level pseudo-algorithms for passive AoA-Doppler estimation.

Input: Snapshot $\mathbf{y}$, f_s
Output: Spectrogram
1 **for** $\theta = 0 : \frac{\pi}{N_{Rx}} : \pi$ **do**
2 | $\mathbf{U} \leftarrow \mathbf{A}(\theta)\mathbf{Y}$ (beamformer), Calculate θ^*;
3 **end**
4 $\hat{N}_{\text{S}}, N_{\text{tmp}} \leftarrow N$; //SBDA starts.
5 **while** $N_{tmp} \leq \hat{N}_S$ **do**
6 | $I(f)$=DFT($N_{\text{tmp}}, \mathbf{y}$), $\mathbf{p}$=findpeaks($I(f)$,rel_max)
7 | **if** card($\mathbf{p}$)=1 **then** $\hat{N}_{\text{S}} = N_{\text{tmp}}$; exit;
8 | **elseif** card($\mathbf{p}$)=2 **then** $N_{\text{tmp}} \leftarrow N_{\text{tmp}} - 1$;
9 **end**
10 Estimate $\hat{A}[n], \hat{\phi}[n]$ //Perform phase correction.
11 **if** $\hat{\phi}[n] > 90^o$ **then** $\mathbf{z} \leftarrow \exp^{-j\hat{\phi}}\mathbf{y}$;
12 **else** $\mathbf{z} \leftarrow \mathbf{y}$;
13 $\mathbf{z}_R \leftarrow \text{Re}(\mathbf{z})$;
14 $I(f)$=DFT($\hat{N}_{\text{S}}, \mathbf{z}_R$);
15 $\mathbf{p}$=findpeaks($I(f)$,rel_max (dB))
16 **if** card($\mathbf{p}$)=1 **then** f_D=median($\mathbf{p}$) ;
17 **elseif** card($\mathbf{p}$)=2 **then** f_D=mean($\mathbf{p}$) ;
18 $ADP(\theta^*, f) = I(f)$;

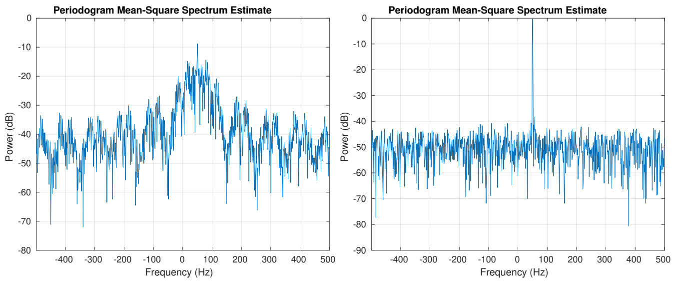
Wireless User Separation with AoA Beamforming: The algorithm starts when a signal is detected on the medium and we collect samples until the signal becomes absent. For the next step the collected data snapshot $\mathbf{y}$ should be labeled according to the user that it belongs. Here, we assume that a specific snapshot belongs to a unique user if the AoA is unique. To estimate its AoA the received signal $\mathbf{y}$ is processed with the AoA Bartlett beamformer [5] for each candidate AoA θ that the resolution of the ULA allows, and this is illustrated in lines 1-3 of the pseudo-Algorithm 1. The AoA that maximizes the beamformer output is indicated as θ^*.

Symbol Boundary Detection Algorithm (SBDA): Symbol boundaries are then detected by identifying phase transitions in the received signal from the DTFT or the periodogram. There is no need for array processing here, that is the data from a single antenna are enough. The SBD algorithm is presented in lines 4-9 of pseudo-Algorithm 1. Recall that the general formula for the periodogram of a dataset $\mathbf{y}$ of N samples is:

$$I(f) = \frac{1}{N}\Big|\sum_{n=0}^{N-1} y[n]e^{-j2\pi fn}\Big|^2 \tag{6}$$

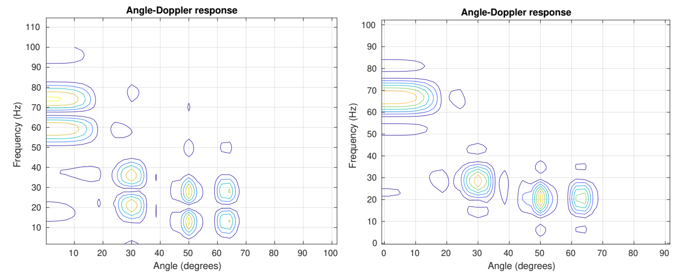
With our algorithm this periodogram is re-calculated repeatedly with different number of samples from $\mathbf{y}$ until a single frequency peak at location $\hat{f}_0$ is present in the spectra. In this way we know, according to our discussion in the last section, that we have no phase transition within the data and so the used samples belong to a single symbol. As we illustrate in pseudo-Algorithm 1 each time $I(f)$ is calculated we reduce the number of samples by one. More efficient strategies can be used but this is not of primary concern to us since SBDA is executed once for multiple snapshots (symbol duration does not change typically in a wireless physical layer frame of a standard like WiFi,LTE,etc.). Due to the finite size of the data record there are sidelobes around $\hat{f}_0$ and there is a need to define a threshold for separating the main lobe from them. This is something that is accounted for in pseudo-Algorithm 1 through the user controlled variable rel_max. Other peaks for which the amplitude of $I(f)$ is more than rel_max dB from the peak at $\hat{f}_0$ are ignored by the findpeaks() function. The cardinality of the set of peaks $\mathbf{p}$ is returned from findpeaks(). The final result of SBDA is an estimate of the number of used samples denoted as $\widehat{N}_{\text{S}}$.

Phase Correction: Once the duration of the symbol in terms of samples has been estimated to be $\widehat{N}_{\text{S}}$ we have to find for each one of the symbols that is included in the snapshot



(a) Without SDBA. (b) With SDBA.

Fig. 3: Periodogram with a receiver SNR of 20dB.



(a) Without SDBA. (b) With SDBA.

Fig. 4: AoA-Doppler Response.

what is the polarity of the data signal $x[n]$. Here we use the ML estimator from example 7.16 in [6], for estimating the amplitude and phase for a signal model like the one in (1). The polarity of the transmitted symbol is actually contained in the phase of the signal model given in (1). So we have that:

$$\widehat{|hx[n]|} = \frac{2\sqrt{I_Y(\hat{f}_0)}}{\hat{N}_\mathrm{S}}, \hat{\phi}[n] = \arctan \frac{\sum_{n=0}^{N-1} y[n] \sin(2\pi \hat{f}_0 n)}{\sum_{n=0}^{N-1} \cos(2\pi \hat{f}_0 n)} \tag{7}$$

Once the phase has been estimated the phase of the corresponding fraction of the data snapshot is reversed as follows:

$$\begin{aligned} z[n] &= e^{-j\hat{\phi}[n]} y[n] \\ &= |hx[n]| e^{j\pi 2(f_D + f_\mathrm{CFO})n/f_s + j(\phi - \hat{\phi})} + e^{-j\hat{\phi}[n]} w[n] \end{aligned} \tag{8}$$

after phase correction in lines 10-12 of the pseudo-Algorithm, the phase of the modulated data has been removed from the signal $\mathbf{y}$ and the resulting samples are contained in $\mathbf{z}$. The periodogram is calculated again for this signal that now contains only the Doppler effect and not any modulation/data-dependent frequency and phase offsets. After DFT, findpeaks() finds the highest peak and then searches again for other peaks within a margin of rel_max dB relative to the maximum, and declares this as the set of identified peaks. Clearly, rel_max has to be empirically set (can investigate see how it should be set up from our evaluation). Depending on the number of peaks we can calculate f_D. The final result is the AoA-Doppler profile $ADP(\theta^*, f) = I(f)$ for the angle θ^* which corresponds to the data vector $\mathbf{y}$. For subsequent snapshots of new PHY frames we repeat the same process (estimate the AoA and the periodogram). Note that when the AoA is the same in subsequent frames, we do not identify this transmission as a new user but update the AoA-Doppler profile (since the speed may have changed).

IV. Performance Evaluation

Results for SDBA: For our evaluation of SDBA we consider a carrier frequency $f_c = 10$GHz, symbol rate of 10^2sps (for easier illustration), $f_s = 10^3$Hz. We set a transmitter moving with 10m/s while the URx is static. Note that the SBDA works on one snapshot and so it can also calculate $I(f)$ (it is equivalent to considering one pulse P=1). In our results we notice in Fig. 3(b) that for an SNR of 20dB the produced spectrogram with our algorithm is considerably better when compared to the baseline system in Fig. 3(a) that calculates it directly from the data vector $\mathbf{y}$. Lower SNR, as expected, reduces the quality of the spectrogram for both cases but with the proposed method the calculation of $I(f)$ is more robust to the same noise level (the peak occurs already around 60dB higher than the sidelobes). It is clear that with the proposed method we only have to face the noise floor.

Performance of the Complete Algorithm: In this part of our evaluation we randomly placed 4 users operating in in an area of 100x100m. Starting from the smaller angle their speeds were 20,10,10,15 m/s to represent different users. Our URx was equipped with N_Rx=10 antennas. A representative AoA-Doppler profile without SDBA and with SDBA is illustrated in Fig. 4. The inability of the system without SDBA to calculate pairs of sources and the Doppler shift is evident. In particular without SDBA the periodogram gives two peaks for a given AoA. Now the need of differentiation of users across different AoA can be seen also for the two users that have a Doppler of 20Hz and are located at AoAs of 50^o and 62^o. Without AoA differentiation these users experience the same Doppler which means that with a typical blind URx their data will be processed as belonging to one source. Our system is able to avoid this. A detail here is that these two users that have the same Doppler do not move at the same speed v, since the Doppler is given by $f_D = \frac{v}{\lambda}\cos(\xi)$ (ξ is the angle between the speed vector of the wireless source and the vector that connects the source to the ULA). Since they may have different ξ their speed v is different according to the previous well known formula.

V. Conclusions

In this paper we investigated the problem of AoA-Doppler profile estimation when the wireless receiver is only aware that a signal is digitally modulated. The proposed system detects symbol transitions and reverses the phase shifts caused by digital modulation. The end result is a higher quality estimate of the periodogram and the AoA-Doppler profile in a wireless environment with multiple sources. The proposed method has applications in passive RADAR systems where the transmitters are uncooperative and information regarding their behavior must be extracted like their AoA and/or Doppler/speed.

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