

Modeling the Lossy Transmission of Correlated Sources in Multiple Access Fading Channels

Antonios Argyriou, and Özgü Alay, and Panagiotis Palantas

Abstract

In this paper, we develop accurate distortion models for the lossy transmission of two correlated sources in a multiple access Rayleigh fading channel. We focus on a class of real-life communication systems, where the source and channel coders have already been designed separately and can only be configured during the system operation. We investigate three different source coding schemes: distributed source coding (DSC), layered source coding, and independent compression through quantization. With the later scheme the sources are jointly decoded with minimum mean square error (MMSE) estimation at the receiver. We also consider two different transmission schemes: Orthogonal transmissions and interfering transmissions decoded with a successive interference cancellation (SIC) decoder. Our final closed-form analytical models are used to determine the optimal combination of source coding and transmission schemes, as well as their optimal configuration. Hence, we exercise joint source and channel coding (JSCC) by optimizing the system configuration. Through simulations, we first validate the analytical model and illustrate the performance of different schemes. Finally, we demonstrate the JSCC gains achieved by our system.

Index Terms

Correlated data, multiple access channel, interference cancellation, Rayleigh fading, wireless sensor networks, distributed source coding, performance model, optimization.

I. Introduction

The most well-known example of correlated data sources are sensors, like video cameras, that collect observations correlated in space and time. The data are typically collected with the help of a wireless sensor

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network (WSN). To reduce the communication bandwidth in such a system, increase robustness to channel errors, and eventually improve the estimation accuracy of the source signal, all the source and channel coding options should be explored. In a traditional lossless communication system, source and channel coding are treated separately. This design choice is based on Shannon's famous theorem that states that the separation of source and channel coding is asymptotically optimal for lossless communication in the point-to-point channel [1]. However, typical WSNs consist of multiple sensors and a single sink/destination that effectively create a multiple access channel (MAC) (Fig. 1). It is well known from the famous result by Cover et al. [2] that performing source and channel coding independently is clearly not optimal for the MAC.

A. Related Work

The potential benefits of joint source-channel coding (JSCC) for MACs have been investigated in the literature, primarily from the information theory community. The literature on the topic contains several works both on the lossless and lossy communication of correlated sources over a MAC [2]–[7]. The previous works treat the topic from the information-theory perspective, i.e., by studying primarily the mutual information that can be attained between the source and the destination under different JSCC schemes, or in certain cases they study the design of codes that are optimal under a JSCC criterion. In [2], the authors proposed a technique, where two sources transmit channel codewords that depend probabilistically on the source data. This means that the correlation of the two sources is inherited in the generated codewords. For two correlated sources, this approach for lossless communication was shown to be superior to a scheme that applies independently the optimal source coding strategy through Slepian-Wolf coding, and the optimal channel code for the MAC. In more recent works on lossless communication, Rajesh et al. in [4] considered a different system model, where side information is available at the sources and the destination. The authors calculated the mutual information between the sources and the destination, and finally derived the required source coding rate subject to side information available at the destination. In [5] the authors investigated lossless and lossy JSCC, and they showed that the optimality of separation holds when correlated side information is available at the receiver. The potential to design a generalized lossless JSCC scheme under a MAC was studied by Yang et al. in [6] but without considering encoding and decoding complexity.

More practical research efforts that are concerned with the problem of transmitting correlated data over the MAC exist, but are limited. One work is a turbo-like JSCC scheme that was proposed by Zhao et al. [8] for the case of two correlated sources transmitted over a fading channel. In that work the authors utilized

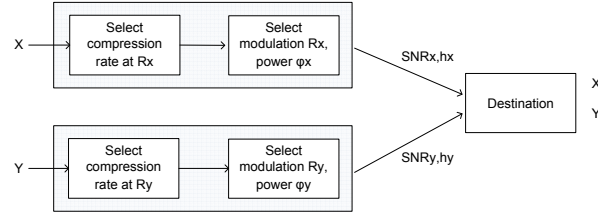


Fig. 1. Two terminals communicate two correlated sources over a multiple access channel (MAC).

orthogonal transmissions, which means that in practice the optimal transmission strategy for the MAC is not used [9], while the encoders at the two sources employed a specific turbo-like structure. A similar idea based on Turbo equalization appeared in [10]. In [11] Banerjee et al. studied the lossless transmission of correlated data with the Slepian-Wolf scheme for a MAC with orthogonal multiple access and also CDMA. Other research avenues for correlated data transmission over a MAC move to the higher layers of the protocol stack without considering the benefits of JSCC [12].

B. Open Issues

One notes that all the above systems were not concerned with their practical implementation. Even if it is possible to apply the optimal source coding strategies for correlated data such as DSC at the application layer [13], the rigid structure of the wireless communication protocol stack does not allow us to fully exploit JSCC in MACs. The reality is that even simple sensor systems that could fully customize the protocol stack, keep the source and channel coding algorithms of the communication system separate. This is mainly due to two specific problems. First, the channel coding algorithm that is employed in a typical transceiver depends on the communication standard, hence, it cannot be modified in a JSCC-optimal fashion. Furthermore, channel coding is typically optimized for the point-to-point channel. Second, data samples from every source in the wireless network are placed in a packet and are transmitted with the help of a multiple access protocol that ensures orthogonal access to each specific source/transmitter. This approach is employed in order to simplify the design of the physical layer (PHY) receiver so that the demodulation algorithm requires only linear processing [9]. Thus, even though there are potential gains through JSCC for correlated data transmission in a MAC, due to practical constraints that originate from the idea of orthogonal channel access and the wide adoption of the separation theorem in hardware, JSCC gains are not fully extracted in practice today.

However, we notice that the parameters of the source coding and channel coding sub-systems are still configurable by the user. We illustrate that in Fig. 1, where we only consider two terminals for exposition purposes. For such a system, the sampling rate and quantizer at the ADC control the compression rate, while in the case of a video encoder more advanced algorithms come into play. Similarly, the channel coding rate, the modulation scheme, and the transmission power control the channel coding. Thus, we argue that there is a need to consider JSCC for a MAC in the context of already implemented but configurable communication systems.

In our previous work [14], we showed that for two compressed correlated sources, the choice of orthogonal transmissions is suboptimal compared to interfering transmissions decoded with successive interference cancellation (SIC) receiver. In that paper, we only considered the use of SIC for increasing the channel capacity without considering compression and estimation schemes that exploit correlation. Still, the benefits were important in terms of the MSE distortion and they were consistent with the expectations we have from all the works we briefly analyzed. Furthermore, the results in our previous work were based on exhaustive numerical evaluation of different configurable source and channel coding parameters.

C. Paper Methodology and Contributions

In this paper, we study different combinations of source and channel coding schemes for the transmission of correlated sources over a MAC. We consider different source coding schemes for the analog sources that are initially quantized [15]. First, we study the theoretically optimal source coding strategy for correlated sources, which is DSC. With DSC correlated sources are compressed separately at each terminal in Fig. 1 and are decoded jointly. DSC requires correlation information at the sources and this was shown to be practical for small number of sources [16], [17]. We consider a Wyner-Ziv type setup, where the compressed version of one source is treated as a remote side information for the compression of the other source. For our second source coding scheme the two sources are independently quantized (i.e., redundancy at the sources is not exploited) as it is typically done in a practical system today. For this second case, at the receiver we consider the use of an adaptive MMSE estimator for exploiting data correlation in order to recover the two source signals from the independently compressed sources. Finally, we also consider a novel layered source coding scheme, where one source is compressed using two layers and the second applies DSC w.r.t. the base layer. This scheme was developed with a mindset towards wireless video applications that are characterized by asymmetric channels (e.g. video surveillance systems).

For channel coding we consider that the sources independently apply a capacity-achieving channel code. The sources may transmit simultaneously over the MAC without any modification at the terminals at the expense of a more complex interference canceling decoder at the receiver. Our closed-form distortion models are limited to Gaussian data only for the DSC scheme, but this is not the case for the MMSE-based scheme that considers an arbitrary data source.

The main contributions of this paper are as follows.

- A closed-form average distortion model for the transmission of correlated sources in a fading MAC when the receiver employs interference cancellation and MMSE decoding.
- A model for DSC with joint SIC/DSC decoding. With DSC if one compressed stream does not reach the destination, the other stream is affected and our model precisely quantifies that, i.e., the impact of interference cancellation on lossy side-information.
- A novel joint layered and DSC transmission system that is also analytically modeled similarly with before.
- An optimization framework JSCC in MACs, where the communication systems can only be configured.

This paper is organized as follows. Our system model is introduced in Section II. We review the effect of link losses on the distortion under distributed compression and MMSE estimation in Section III. Section IV presents the outage analysis for all the communications schemes. The combination of the results of the last two sections in our final model is presented and the optimization problem in Section V. Section VI presents the performance of the proposed strategies for different source and channel conditions, while the case of layered sources is analyzed in Section VII. We conclude the paper in Section VIII.

II. System model

We consider a system, where T_x and T_y are two terminals in a wireless network communicating with a common destination. We assume terminals T_x and T_y have access to two correlated sources X and Y , respectively, which they wish to transmit to the destination with minimal expected MSE distortion over several channel realizations. Without loss of generality, we can write, $Y = aX + Z$ when X and Y are jointly Gaussian and correlated. Here $Z \sim \mathcal{N}(0, \sigma_z^2)$ is independent of X with $\sigma_z^2 = \sigma_y^2 - a^2\sigma_x^2$ and $a = \rho \frac{\sigma_y}{\sigma_x}$. The covariance matrix of the two sources

$$K_{XY} = \begin{bmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{bmatrix}, \quad (1)$$

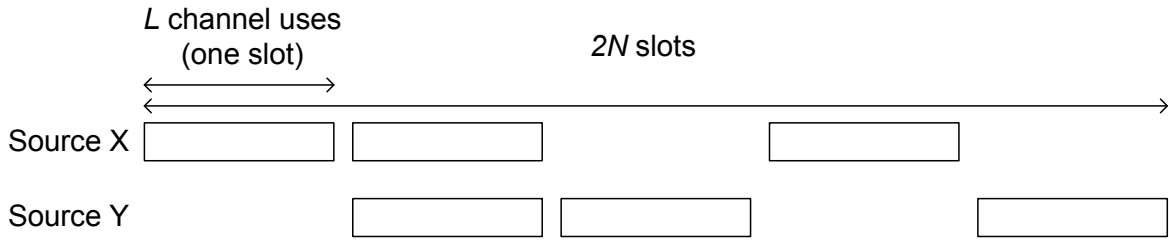


Fig. 2. Data sample transmissions according to our system model. During a fraction β of the $2N$ slots the two sources transmit simultaneously and interfere.

where ρ is the correlation coefficient, i.e., this is a memoryless Gaussian bivariate source.

At each time slot that consists of L channel uses, each source sends K source samples. For T_x , we assume the number of transmitted information bits (or source bits) per channel use (bpcu) is R_x . This results in a compression rate of $\bar{R}_x = \frac{LR_x}{K} = bR_x$ bits per source sample. The compression results in terms of distortion depend on the adopted source coding scheme and this is the main focus of the analysis for the next sections. But regardless of the compression scheme, the digital observation for one sample can be written as

$$Y_d = \alpha X + Z + Q_y \text{ and } X_d = X + Q_x. \quad (2)$$

In the above Q_x, Q_y are the samples of the quantization noise that have variance equal to $D_x(\bar{R}_x)$ and $D_y(\bar{R}_y)$, respectively. The K digitized samples are then digitally modulated into packets of size LR_x and LR_y bits for the two sources. Finally, each packet from the two sources X, Y is channel-coded with a capacity-achieving AWGN code of LR_x, LR_y bits, respectively, and transmitted within the time slot defined by the system. We assume that a complete packet will be discarded if the channel decoder can not correct all the errors, again in accordance with the behavior of real wireless transceivers.

Communication is carried over two links that have flat Rayleigh fading with instantaneous fading levels h_x and h_y (i.e., $h_x, h_y \sim \mathcal{CN}(0, 1)$), and average received signal to noise ratios $SNR_x = \frac{\varphi_x}{N_0}$ and $SNR_y = \frac{\varphi_y}{N_0}$, where φ_x and φ_y are the transmit powers for T_x and T_y , respectively. The fading levels are accurately measured at the receiver (only required for coherent demodulation), while the transmitters do not need to be aware of any aspect of the channel or its statistics. The fading is assumed to change across the different time slots.

For the transmission of the modulated digital packets of the two correlated sources over the Rayleigh channel, we assume there are in total $2N$ slots available (Fig. 2) and we consider two different basic transmission options. First, we assume the sources are transmitting orthogonally using time division multiple access (TDMA). In this case, the baseband received signal at the receiver is

$$r_y = h_y \sqrt{\varphi_y} Y_d + W \text{ and } r_x = h_x \sqrt{\varphi_x} X_d + W, \quad (3)$$

where W is the AWGN sample at the receiver with variance σ_w^2 . With the second scheme, we allow T_x and T_y to transmit at the same time, hence, allowing interfering transmission. In this case both terminals utilize the total transmission time of a slot.¹ Assuming T_x and T_y are each utilizing N slots with an orthogonal TDMA-based scheme, with interfering transmissions they would transmit for $2N$ slots. Note that, these are two extreme cases in terms of usage of the slots. In this paper, we define β to be the fraction of time, where the two sources interfere. This allows us to model and evaluate the performance of all different cases as illustrated in Fig. 2.

To ensure a fair comparison of all the tested schemes all source transmissions are constrained to a specific power level per each time slot. This means that when orthogonal transmissions are used, then the source transmits with power φ , while for the schemes that use interfering transmissions the total transmission power of both sources is split as $\varphi_x + \varphi_y = \varphi$.

Finally, we assume that the two sources are fully synchronized with the receiver in the following sense [18]: First, there is carrier synchronization, i.e., both sources have a local oscillator synchronized to the receiver carrier frequency; Second, regarding time synchronization and each channel use, the relative timing error between the sources transmissions is much smaller than the channel symbol duration T_c .

A. Transmission Schemes

In order to illustrate the effects of interfering transmission on correlated sources, we consider different source coding and transmission schemes.

- ORTH-DSC: This mode utilizes only distributed compression. We study a specific scheme, where Y is compressed independently according to the rate distortion bound and X is compressed based on Y , but in a robust fashion realizing that the compressed version of Y may not be available at the destination.

¹When interference is allowed the source transmissions interfere for the full duration of one slot

- ORTH-MMSE: Each terminal compresses and transmits its own source directly to the destination in its own timeslot. The terminals ignore the source correlation but correlation is extracted at the receiver with MMSE estimation. The details will be discussed in Section III-B.
- SIC-DSC: In this mode, distributed compression along with interference decoding under SIC is considered. The terminals transmit the correlated sources allowing interference and by employing DSC.
- SIC-MMSE: In this mode interfering transmissions are allowed. The sources are compressed independently and the source correlation is extracted at the receiver with MMSE estimation.
- Finally, we examine novel combination of DSC with layered source coding of a single source. The scheme is named DSC-LS or DSC-BS depending on the specific flavor of the layered source coding scheme that we use.

III. Expected end-to-end Distortion Calculation with a Single Layer

In this section we formulate the expected distortion for different schemes. Due to the relative complexity nature of the expressions of layered source coding we develop that scheme in a separate section. Now given the knowledge of decoded packets, then the distortion expressions only depend on the source coding scheme and its configuration. Therefore, in this section, we group the modes into two in terms of the way the sources are compressed. However, while computing the expected distortion expressions, the outage probabilities will be considered. The outage probabilities for different transmission strategies (orthogonal vs. interfering transmissions) will be derived analytically in the next section, Section IV.

A. Joint Decoding of Independently Compressed Sources

We first investigate the distortion expressions for the independent source compression and MMSE decoding. Individual source compression is exercised through uniform probabilistic quantization which means that the distortion of the Gaussian source X is:²

$$D_x(\bar{R}_x) = \frac{W^2}{(2\bar{R}_x - 1)^2}. \quad (4)$$

In the above, $2W$ is the range of the sensed signal.

In this case we exploit knowledge of the correlation model between sources X, Y at the receiver through joint linear MMSE decoding. Alternatively, when the correlation model is unknown, one can employ the Weighted Least Square (WLS) estimator. Nevertheless, the fundamental choice is the correlation exploitation

²Note that under MMSE decoding, any general quantization scheme and source distribution can be supported.

at the receiver. Note that this also means higher transmission bandwidth when compared to DSC. This is essentially a JSCC tradeoff since the transmission of uncompressed correlated data implies a redundancy to the transmission.

When both digital packets X_d, Y_d are correctly decoded at the receiver, then we notice that we have two decoded digital observations that are correlated. From these two observations we can jointly estimate X with the MMSE approach (recall our data model captured in (3)). This leads to the distortion of X being equal to

$$D_x^{(1)} = \mathbb{E}[(X - \hat{X})^2] = \frac{\sigma_x^2}{\sigma_x^2 \left(\frac{\alpha^2}{\sigma_z^2 + D_y(\bar{R}_y)} + \frac{1}{D_x(\bar{R}_x)} \right) + 1}. \quad (5)$$

The numerical superscript above indicates the first event, i.e., that both packets were decoded. The distortion of Y in this case is

$$D_y^{(1)} = D_y(\bar{R}_y).$$

The reason for the above expression is that the receiver can estimate Y since it has available the digital compressed signal Y_d with distortion $D_y(\bar{R}_y)$.

Let us now consider the second event that X_d is decoded and Y_d is not. Then, the distortion of X is

$$D_x^{(2)} = D_x(\bar{R}_x).$$

Similarly, we estimate source Y as $\hat{Y} = \alpha \hat{X}$ since now we do not have any other observation of this signal.

Thus, the distortion of Y is

$$D_y^{(2)} = \mathbb{E}[(Y - \hat{Y})^2] = \alpha^2 D_x(\bar{R}_x) + \sigma_z^2.$$

Note that if we did not use at all the decoded signal \hat{X} , the distortion for Y would be equal to $\sigma_y^2 = \alpha^2 \sigma_x^2 + \sigma_z^2$ which is clearly the worst case for the distortion of Y .

Now we consider that Y_d is decoded and X_d is not. The distortion for Y is then equal to

$$D_y^{(3)} = D_y(\bar{R}_y).$$

We can also use MMSE estimation for X from our data model in (3) and in this case the distortion is equal to

$$D_x^{(3)} = \frac{\sigma_x^2}{\sigma_x^2 \left(\frac{\alpha^2}{\sigma_z^2 + D_y(\bar{R}_y)} \right) + 1}.$$

What we do in all these cases is first we take advantage the availability of different combinations of observations from SIC decoding, and second we use the knowledge of the data model in order to appropriately estimate each source. Thus we exploit the data correlation for estimating both sources X and Y .

For a given communication mode, average channel SNRs and source correlation, the expected distortion is a function of the source rates and the amount of channel coding. The average distortion expressions for the system that exploits correlation at the receiver can be written as

$$ED_x(a, \sigma_x^2, \sigma_y^2, \sigma_z^2, \bar{R}_x, \bar{R}_y) = P^{(1)}D_x^{(1)} + P^{(2)}D_x^{(2)} + P^{(3)}D_x^{(3)} + P^{(4)}\sigma_x^2. \quad (6)$$

where the probabilities $P^{(i)}$ are defined as the average probability of state (i) , $i \in \{1, 2, 3, 4\}$. i denotes whether (X, Y) is received. For $i = 1$, the compressed bits of both T_x and T_y are received, for $i = 2$ the compressed bits of T_x are received but the bits for T_y are lost, $i = 3$ means the compressed bits of T_y are received but the bits for T_x are lost and finally for $i = 4$ the compressed bits of both T_x and T_y fail to reach the destination. This probability expressions will be calculated in Section IV. The expression for ED_y can be obtained similarly.

B. Distributed Source Coding

Now we compute expected distortions corresponding to the DSC-based system. Finding a general rate-distortion region for distributed compression when the compressed streams may be lost is a challenging problem [19]. That is why we concentrate on the asymmetric scenario, where Y is compressed separately and X is compressed with respect to Y . Also note that for Gaussian sources, having side information at both the source encoder and the decoder does not improve the rate distortion performance over having it only at the decoder [20].

Suppose Y is not available at X 's encoder, and may or may not be available at the X 's decoder. Let D_1 denote the squared error distortion achieved when Y is present at the destination, D_2 denote the distortion achieved when Y is absent. In the DSC setup with unreliable links illustrated in Fig. 3, Y is also compressed, hence, lossy. The rate distortion function of X when lossy side information may be absent is [21]:

$$R(D_1, D_2) = \begin{cases} \frac{1}{2} \ln\left(\frac{\sigma_x^2(\sigma_z^2 + \sigma_w^2)}{D_1(a^2 D_2 + \sigma_z^2 + \sigma_w^2)}\right) & \text{if } D_1 \leq \sigma_2^2, D_2 \leq \sigma_x^2 \\ \frac{1}{2} \ln\left(\frac{\sigma_x^2}{D_2}\right) & \text{if } D_1 \geq \sigma_2^2, D_2 \leq \sigma_x^2 \\ \frac{1}{2} \ln\left(\frac{\sigma_x^2(\sigma_z^2 + \sigma_w^2)}{D_1(a^2 \sigma_z^2 + \sigma_z^2 + \sigma_w^2)}\right) & \text{if } D_1 \leq \sigma_2^2, D_2 > \sigma_x^2 \\ 0 & \text{if } D_1 \geq \sigma_2^2, D_2 > \sigma_x^2 \end{cases} \quad (7)$$

where $\sigma_2^2 = \frac{D_2(\sigma_z^2 + \sigma_w^2)}{a^2 D_2 + \sigma_z^2 + \sigma_w^2}$, $\sigma_w^2 = \frac{\sigma_y^2 D_y}{\sigma_y^2 - D_y}$.

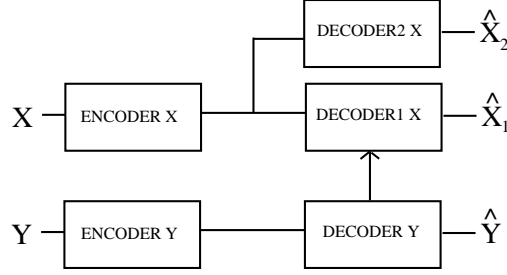


Fig. 3. Compression with DSC when lossy side information may be absent.

We are mainly interested in the compression rate of X in the regime $D_1 \leq \sigma_y^2$, $D_2 \leq \sigma_x^2$. In this region, for a given R_x , D_2 and D_y , the distortion D_1 is equal to

$$D_1(\bar{R}_x, D_2, D_y) = \frac{\sigma_x^2(\sigma_y^2\sigma_x^2 + a^2\sigma_x^2D_y)}{a^2D_2\sigma_y^2 - a^2D_2D_y + \sigma_y^2\sigma_x^2 + a^2\sigma_x^2D_y} 2^{-2\bar{R}_x}$$

where \bar{R}_x is the compression rate of X in bits/sample.

Finally, the side information Y is compressed using \bar{R}_y bits per source sample and is sent directly to the destination with a distortion of:

$$D_y(\bar{R}_y) = \frac{W^2}{(2^{\bar{R}_y} - 1)^2} \quad (8)$$

Hence, the average distortion for the schemes that use DSC, can be calculated as

$$\begin{aligned} ED_x(\sigma_x^2, \sigma_y^2, D_2, \bar{R}_x, \bar{R}_y) &= P^{(1)} D_1(\bar{R}_x, D_2, D_y) + P^{(2)} D_2 \\ &+ P^{(3)} D_1(0, D_2, D_y) + P^{(4)} \sigma_x^2. \end{aligned} \quad (9)$$

$$ED_y(\sigma_y^2, \bar{R}_y) = (P^{(1)} + P^{(3)})D_y(\bar{R}_y) + (P^{(2)} + P^{(4)})\sigma_y^2 \quad (10)$$

In the above formulation, D_y is given by (8). Also recall D_2 is the target distortion for X if the description of Y is lost. Note that in (9) and for different decoding events, we have a different average distortion and this makes the expression more complicated than the systems that do not use DSC.

IV. Outage Analysis for Rayleigh Fading

To compute the average outage probabilities $P^{(i)}$ for the wireless transmissions, we proceed as follows. Considering complex Gaussian codebooks, for a channel code operating at a rate R bpcu, information is lost when the instantaneous channel capacity is lower than R , leading to the outage probability $P_{out} = \Pr\{C(|h|^2SNR) < R\}$ for a point to point link, where $C(x) = \log_2(1+x)$ is the Gaussian channel capacity

and $|h|$ is the fading amplitude. Note that the outage expression is an averaging over several transmissions of codewords of R bits, i.e.,

$$P_{out} = \Pr\{C(|h|^2 SNR) < R\} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1, \dots, N} \mathbf{1}_{\{C(|h(i)|^2 SNR) < R\}} \quad (11)$$

By utilizing the previous approach, and as an example, we will illustrate the computation of $P^{(1)}$ which is the probability that both sources are decoded. The channel capacity under the interference and the orthogonal transmission cases can be computed by looking into the transmission pattern of our system model (Fig. 2). Recall that in this paper we defined β as the fraction of the time (over the $2N$ channel uses) that the two sources interfere. We can write the probability of receiving both X and Y for a specific value of β as

$$\begin{aligned} P^{(1)}(\beta) &= \beta \Pr\{R_x < C(SINR_x), R_y < C(SINR_y)\} \\ &+ (1 - \beta) \Pr\{R_x < C(|h_x|^2 SNR_x), R_y < C(|h_y|^2 SNR_y)\}. \end{aligned} \quad (12)$$

This expression is the probability that both sources are successfully decoded if they interfere for a fraction β of the time during the $2N$ slots and as $N \rightarrow \infty$.

To understand how we obtained this expression consider the extreme case that $\beta=0$, meaning each source accesses the channel for NL channel uses (out of the total $2NL$). Then, the outage probability $P^{(1)}(\beta = 0)$ for N orthogonal slots is:

$$\begin{aligned} P^{(1)}(\beta = 0) &= \Pr\{R_x < C(|h_x|^2 SNR_x), R_y < C(|h_y|^2 SNR_y)\} \\ &= \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1, \dots, N} \mathbf{1}_{C(|h_x(i)|^2 SNR_x) < R_x, C(|h_y(i)|^2 SNR_y) < R_y} \end{aligned} \quad (13)$$

This corresponds the second probability term in (12). The first probability term is obtained slightly differently. When $\beta=1$ then both sources access the channel for a double number of channel uses and that is why the averaging occurs over $2N$ slots. Thus we have:

$$\begin{aligned} P^{(1)}(\beta = 1) &= \Pr\{R_x < C(SINR_x), R_y < C(SINR_y)\} \\ &= \lim_{2N \rightarrow \infty} \frac{1}{2N} \sum_{i=1, \dots, 2N} \mathbf{1}_{C(|h_x(i)|^2 SINR_x) < R_x, C(|h_y(i)|^2 SINR_y) < R_y} \end{aligned} \quad (14)$$

Note that in the previous expressions we applied our system model constraint of constant power per channel use. That is the transmit power for the interfering sources should satisfy $\varphi = \varphi_x + \varphi_y$. The transmit power for each source appears inside the SINR expressions.

Similar expressions can be produced for $P^{(2)}$, $P^{(3)}$, $P^{(4)}$ by simply changing the inequalities in (12). Now in the next sections we calculate the two probability terms in (12).

Remark 1: The actual number of slots with no interference and interference is $(1 - \beta)N$ and βN , respectively. However, our model is only sensitive to β for finite number of N since N is non-existent in the L.H.S. of (14) (when $N \rightarrow \infty$).

A. Outage Probability for orthogonal transmissions ($\beta=0$)

When the sources do not interfere, the two decoding events are independent, hence, we have:

$$\begin{aligned} P^1(\beta = 0) &= \Pr\{R_x < C(|h_x|^2 SNR_x), R_y < C(|h_y|^2 SNR_y)\} \\ &= \Pr\{R_x < C(|h_x|^2 SNR_x)\} \times \Pr\{R_y < C(|h_y|^2 SNR_y)\} \\ &= \left(1 - \exp\left(-\frac{2^{R_x} - 1}{SNR_x}\right)\right) \left(1 - \exp\left(-\frac{2^{R_y} - 1}{SNR_y}\right)\right) \end{aligned} \quad (15)$$

Similarly we can compute the probability $P^{(i)}$ of the other events.

B. Outage Probability for Interfering Transmissions with SIC ($\beta=1$)

The signal model that is used for the case of interference is:

$$I = \sqrt{\varphi_x} h_x X_d + \sqrt{\varphi_y} h_y Y_d + W$$

where X_d and Y_d are the digital transmitted packets.

Here we assume an ordered SIC (OSIC) decoder is used which means that the stronger signal is decoded first. If there is no interference the following condition must be true so that the digital packet from source X is decoded:

$$\log_2\left(1 + \frac{\varphi_x |h_x|^2 \sigma_x^2}{\sigma_w^2}\right) \geq R_x \Rightarrow \frac{\varphi_x |h_x|^2 \sigma_x^2}{\sigma_w^2 (2^{R_x} - 1)} \geq 1$$

The fractional term in the R.H.S. of the last derivation is essentially the normalized SNR/bit that is required for decoding R_x bits/symbol [22]. We can get a similar expression for Y .

For exposition purposes, let us assume that the stronger signal (higher SNR/bit or energy/bit) is X . Then, the symbols from X are decoded first only if the condition is met:

$$\frac{\varphi_x |h_x|^2 \sigma_x^2}{\sigma_w^2 (2^{R_x} - 1)} > \frac{\varphi_y |h_y|^2 \sigma_y^2}{\sigma_w^2 (2^{R_y} - 1)}, \quad (16)$$

In this case, the Signal to Interference plus Noise Ratio (SINR) for X_d can be expressed as

$$SINR_x = \frac{\varphi_x |h_x|^2 \sigma_x^2}{\varphi_y |h_y|^2 \sigma_y^2 + N_0}. \quad (17)$$

Now, after decoding X_d , we can decode Y_d with its respective SINR being equal to

$$SINR_y = \frac{\varphi_y |h_y|^2 \sigma_y^2}{\sigma_w^2}. \quad (18)$$

We can now write the probability of receiving both X_d and Y_d for the case of interfering transmissions according to (14). Calculating (14) in a simulation setup through averaging is a straightforward task [14]. This could be accomplished with the averaging formulas we presented earlier. However, there are complications in calculating analytically the previous joint outage probability for the two sources. First, the $SINR_x, SINR_y$ expressions we provided above are conditioned on which signal was decoded first (if Y was stronger then the expressions would have the opposite form). Second, packet decoding events are not independent which means that this probability cannot be decoupled as in (15).

Thus, our goal is to calculate the probability expression in (14) analytically but for all the potential decoding outcomes (i.e., $P^{(2)}(\beta = 1)$ etc.). For minimizing the complexity of our derived expressions, we define the following random variables along with their expectation: $U = \varphi_x |h_x|^2 \sigma_x^2, V = \varphi_y |h_y|^2 \sigma_y^2, \mathbb{E}[|h_x|^2] = \frac{1}{\mu}, \mathbb{E}[|h_y|^2] = \frac{1}{\lambda}$. In the remainder of this section, we only manipulate exponential random variables. It is also important to clarify that in our analysis we also consider the impact of error propagation in SIC, which is a critical piece that affects its performance in practice [23].

C. Outage Probability of Joint Events

a) Both sources are in outage: We first consider the event, where both sources are simultaneously in outage when SIC is applied. This is defined as

$$P^{(4)} = \Pr(I_X < R_x, I_Y < R_y). \quad (19)$$

where I_X, I_Y indicate the mutual information between the signals transmitted from the sources X and Y, respectively, and the received signals at the destination. When we apply OSIC, I_X will be different depending which signal has the highest energy/bit since this will be the one that will be decoded first. In particular, if the signal from X is stronger in terms of energy/bit than the signal from Y, i.e., if $U > \frac{k_x}{k_y} V$ (condition (16)), then, we have:

$$I_X = \log_2 \left(1 + \frac{U}{V + \sigma_w^2} \right). \quad (20)$$

where $k_x = 2^{R_x} - 1, k_y = 2^{R_y} - 1$ are the SNR packet decoding thresholds, again in our effort to minimize notation in the remaining of this paper.

When the signal from source Y has higher energy/bit than source X, then OSIC will first decode source Y and remove it from the aggregate, leading to

$$I_X = \log_2\left(1 + \frac{U}{\sigma_w^2}\right). \quad (21)$$

By considering the behavior of SIC, we further notice that the event in (19) can be decomposed to two mutually exclusive events depending on which signal is decoded first:

$$\begin{aligned} \Pr(I_X < R_x, I_Y < R_y) &= \Pr(I_X < R_x, I_Y < R_y, U > \frac{k_x}{k_y}V) \\ &+ \Pr(I_X < R_x, I_Y < R_y, U < \frac{k_x}{k_y}V) \end{aligned} \quad (22)$$

Substituting (20) and (21) and elaborating on the last equation, we have:

$$\begin{aligned} \Pr(I_X < R_x, I_Y < R_y) &= \Pr(U - k_x V < k_x \sigma_w^2, U > \frac{k_x}{k_y}V) \\ &+ \Pr(V - k_y U < k_y \sigma_w^2, U < \frac{k_x}{k_y}V) \end{aligned} \quad (23)$$

The event $U > \frac{k_x}{k_y}V$ considers all the cases where the packet, that originates from X, has the highest energy/bit and thus it will be decoded first. If this last event is true, then $U - k_x V < k_x \sigma_w^2$ is the event that source X cannot be decoded (event $I_X < R_x$). Note that this joint event includes the case that source Y cannot be decoded if $U > \frac{k_x}{k_y}V$. The reason is simply that the energy/bit is lower for source Y and so if source X cannot be decoded we can definitely not decode source Y. This is precisely the impact of error propagation in SIC.

To calculate (23) we have to recall that U and V are independent exponential random variables. This means that their joint probability density function (PDF) is separable. Thus, the first event is calculated as

$$\begin{aligned} &\Pr(U - k_x V < k_x \sigma_w^2, U > \frac{k_x}{k_y}V) \\ &= \int_0^\infty f_V(v) \int_{\frac{k_x}{k_y}v}^{k_x v + k_x \sigma_w^2} f_U(u) du dv \\ &= \frac{\lambda}{\lambda + \mu \frac{\varphi_y \sigma_y^2}{\varphi_x \sigma_x^2} \frac{k_x}{k_y}} - \frac{\lambda \exp(-\frac{\mu}{\varphi_x \sigma_x^2} k_x \sigma_w^2)}{\lambda + \mu \frac{\varphi_y \sigma_y^2}{\varphi_x \sigma_x^2} k_x}. \end{aligned} \quad (24)$$

Similarly we calculate the probability of the second event in (23) and then by adding the two results we finally obtain:

$$\begin{aligned}
P^{(4)} &= \Pr(I_X < R_x, I_Y < R_y) \\
&= \frac{\lambda}{\lambda + \mu \frac{\varphi_y \sigma_y^2}{\varphi_x \sigma_x^2} k_x / k_y} - \frac{\lambda \exp(-\frac{\mu}{\varphi_x \sigma_x^2} k_x \sigma_w^2)}{\lambda + \mu \frac{\varphi_y \sigma_y^2}{\varphi_x \sigma_x^2} k_x} \\
&+ \frac{\mu}{\lambda \frac{\varphi_x \sigma_x^2}{\varphi_y \sigma_y^2} k_y / k_x + \mu} - \frac{\mu \exp(-\frac{\lambda}{\varphi_y \sigma_y^2} k_y \sigma_w^2)}{\mu + \lambda \frac{\varphi_x \sigma_x^2}{\varphi_y \sigma_y^2} k_y}
\end{aligned}$$

b) X is in outage and Y is not in outage: With a similar methodology we calculate the outage probability of the second event that considers the case that X in outage while Y is not:

$$\begin{aligned}
P^{(3)} &= \Pr(I_X < R_x, I_Y > R_y) \\
&= \Pr(I_X < R_x, I_Y > R_y, U > \frac{k_x}{k_y} V) \\
&+ \Pr(I_X < R_x, I_Y > R_y, U < \frac{k_x}{k_y} V)
\end{aligned}$$

In the last decomposed expression we followed the similar procedure with before. Only in this case the probability of the first of the two disjoint events is zero. This is again a result of the behavior of OSIC that selects to decode first the symbol with the highest energy/bit. If the signal from source X is the stronger ($U > \frac{k_x}{k_y} V$), and the destination fails to decode it ($I_X < R_x$), then it is impossible to decode source Y. This leads to

$$\begin{aligned}
P^{(3)} &= \Pr(I_X < R_x, I_Y > R_y, U < \frac{k_x}{k_y} V) + 0 \\
&= \Pr(U < k_x \sigma_w^2, V - k_y U > k_y \sigma_w^2, U < \frac{k_x}{k_y} V) \\
&= \frac{\mu \exp(-\frac{\lambda}{\varphi_y \sigma_y^2} k_y \sigma_w^2)}{\mu + \lambda \frac{\varphi_x \sigma_x^2}{\varphi_y \sigma_y^2} k_y} \left(1 - \exp(-(\frac{\mu}{\varphi_x \sigma_x^2} + \frac{\lambda}{\varphi_y \sigma_y^2} k_y) k_x \sigma_w^2) \right).
\end{aligned} \tag{25}$$

In the above note that when the decoding of Y succeeds, then the signal will be removed from the aggregate. This means that the event that X is not decoded is $U < k_x \sigma_w^2$ since the decoder must only combat the noise after cancellation.

c) Y is in outage and X is not in outage: The third event is symmetric to what we just analyzed: The second source Y is in outage, while the first source X is successfully decoded. This is expressed as

$$\begin{aligned}
P^{(2)} &= \Pr(I_X > R_x, I_Y < R_y) \\
&= \Pr(U - k_x V > k_x \sigma_w^2, V < k_y \sigma_w^2, U > \frac{k_x}{k_y} V) \\
&+ \Pr(I_X > R_x, I_Y < R_y, U < \frac{k_x}{k_y} V).
\end{aligned} \tag{26}$$

With the reasoning we followed in the previous paragraphs we can easily see that the probability of the second of the two events above is zero. Thus, (26) becomes

$$\begin{aligned}
&\Pr(I_X > R_x, I_Y < R_y) = \\
&\Pr(U - k_x V > k_x \sigma_w^2, V < k_y \sigma_w^2, U > \frac{k_x}{k_y} V) + 0
\end{aligned} \tag{27}$$

Since the integration is slightly more complicated in this case we calculate the previous expression as

$$\begin{aligned}
&\Pr(U - k_x V > k_x \sigma_w^2, V < k_y \sigma_w^2, U > \frac{k_x}{k_y} V) \\
&= \Pr(X < k_y \sigma_w^2, U > \frac{k_x}{k_y} V) \\
&- \Pr(U - k_x V < k_x \sigma_w^2, V < k_y \sigma_w^2, U > \frac{k_x}{k_y} V).
\end{aligned}$$

And the calculation gives:

$$\begin{aligned}
P^{(2)} &= \Pr(U - k_x V > k_x \sigma_w^2, V < k_y \sigma_w^2, U > \frac{k_x}{k_y} V) \\
&= \int_0^{k_y \sigma_w^2} f_V(v) \int_{\frac{k_x}{k_y}}^{\infty} f_U(u) du dv - \int_0^{k_y \sigma_w^2} f_V(v) \int_{\frac{k_x}{k_y}}^{k_x v + k_x \sigma_w^2} f_U(u) du dx \\
&= \frac{\lambda \exp(-\frac{\mu}{\varphi_x \sigma_x^2} k_x \sigma_w^2)}{\lambda + \mu \frac{\varphi_y \sigma_y^2}{\varphi_x \sigma_x^2} k_x} \left(1 - \exp\left(-\left(\frac{\lambda}{\varphi_y \sigma_y^2} + \frac{\mu}{\varphi_x \sigma_x^2} k_x\right) k_y \sigma_w^2\right) \right)
\end{aligned} \tag{28}$$

d) Both X and Y are not in outage: Finally, we consider the event that both sources are not in outage. Since all the four events that we described in the last few paragraphs are mutually exclusive, and these

$$\begin{aligned}
ED_x^{\text{MMSE}} &= P^{(1)}(\lambda, \mu, R_x, R_y, \varphi_x, \varphi_y, N_0, \beta) D_x^1(\rho, \sigma_x^2, \sigma_y^2, \bar{R}_x, \bar{R}_y) \\
&+ P^{(2)}(\lambda, \mu, R_x, R_y, \varphi_x, \varphi_y, N_0, \beta) D_x^2(\bar{R}_x) \\
&+ P^{(3)}(\lambda, \mu, R_x, R_y, \varphi_x, \varphi_y, N_0, \beta) D_x^3(\rho, \sigma_x^2, \sigma_y^2, \bar{R}_x, \bar{R}_y) \\
&+ P^{(4)}(\lambda, \mu, R_x, R_y, \varphi_x, \varphi_y, N_0, \beta) \sigma_x^2
\end{aligned} \tag{30}$$

probability calculations sum up to one, we have:

$$\begin{aligned}
P^{(1)} &= \Pr(I_X > R_x, I_Y > R_y) \\
&= \Pr(U - k_x V > k_x \sigma_w^2, V > k_y \sigma_w^2, U > \frac{k_x}{k_y} V) \\
&+ \Pr(U > k_x \sigma_w^2, V - k_y U > k_y \sigma_w^2, U < \frac{k_x}{k_y} V) \\
&= \frac{\lambda \exp(-\frac{\mu}{\varphi_x \sigma_x^2} k_x \sigma_w^2)}{\lambda + \mu \frac{\varphi_y \sigma_y^2}{\varphi_x \sigma_x^2} k_x} \left(1 - \exp\left(-\left(\frac{\lambda}{\varphi_y \sigma_y^2} + \frac{\mu}{\varphi_x \sigma_x^2} k_x\right) k_y \sigma_w^2\right) \right) \\
&+ \frac{\mu \exp(-\frac{\lambda}{\varphi_y \sigma_y^2} k_y \sigma_w^2)}{\mu + \lambda \frac{\varphi_x \sigma_x^2}{\varphi_y \sigma_y^2} k_y} \exp\left(-\left(\frac{\mu}{\varphi_x \sigma_x^2} + \frac{\lambda}{\varphi_y \sigma_y^2} k_y\right) k_x \sigma_w^2\right) \\
&- \frac{\lambda}{\lambda + \mu \frac{\varphi_y \sigma_y^2}{\varphi_x \sigma_x^2} k_x / k_y} - \frac{\mu}{\mu + \lambda \frac{\varphi_x \sigma_x^2}{\varphi_y \sigma_y^2} k_y / k_x}
\end{aligned} \tag{29}$$

V. Final Model and Optimization

The results we obtained in the previous section lead to a closed form expression for $P^{(1)}(\beta)$ given in (12), while similar expressions are derived for all the other three events. Now the average distortion expressions for the case of interfering transmissions and MMSE estimation can be written by combining (36) and (12) and this leads to the final expression in (30). With the same process we combine the outage and distortion expressions for the different transmission modes and compression schemes that we presented in the last section.

Having derived the distortion model we can directly proceed with the JSCC optimization. In this case we use (30) (and the associated expressions for ED_y) in order to solve numerically for the optimal configuration of the source and channel coding subsystems in terms of \bar{R}_x, \bar{R}_y , and ϕ_x, ϕ_y, R_x, R_y , respectively. Formally,

the problem for an MMSE-based system is:

$$\begin{aligned} \text{JSCC-MAC : } \min_{\bar{R}_x, \bar{R}_y, \beta, \phi_x, \phi_y, R_x, R_y} (ED_x^{\text{MMSE}} + ED_y^{\text{MMSE}}) \\ \text{subject to } \quad \phi_x + \phi_y \leq \phi, \beta \in [0, 1], R_x, R_y \in \mathcal{R} \end{aligned} \quad (31)$$

In the above \mathcal{R} is the discrete set of the available modulation schemes. Note in the above that β (the optimal level of overlap between the transmissions of the two sources) is an optimization parameter. This formulation is a non-linear program that of course does not admit any closed-form solution. However, it is important to clarify that this is an optimization formulation for the average MSE over several channel realizations which means that the optimal solution can be practically enforced regardless of need for a numerical solution. Finally we should note from the DSC-based expressions that the problem formulation similar but in that case we have an additional optimization parameter D_2 .

VI. Performance Results for a Single Layer

In this section, we evaluate the performance of each transmission mode with simulations over a wide range of source correlation coefficients and average channel SNRs. At the same time we verify the validity of the analytical model by plotting numerical results together with the simulations. In the second part of our evaluation, we demonstrate the benefits of the proposed system for JSCC optimization.

The simulation results were obtained by considering 4000 realizations of the Rayleigh fading channel. For each iteration we calculated whether the packet was decoded at the destination and calculated the average number of decoded packets for each source. For MMSE decoding, the MSE was measured by calculating these average number of packet losses, and the well-known analytical MSE distortion expressions for the distortion in a linear observation model that we developed in Section III. For the DSC-based system, we similarly simulated the channel and the packet decoding. The average number of decoded packets measured at the destination was used in conjunction with the DSC analytical formulas.

A. Simulation Results and Model Validation

In this subsection we set $\sigma_x^2 = \sigma_y^2 = 1$, $R_x = R_y = 1$ (BPSK modulation) and $b = \frac{L}{K} = 1$, hence, $\bar{R}_x = R_x$. We plot results for different asymmetric and symmetric channel SNR scenarios expressed through $SNR_x = \frac{\varphi_x}{\sigma_w^2}$ and $SNR_y = \frac{\varphi_y}{\sigma_w^2}$. For our DSC-based simulations in this section there is an extra step needed since D_2 is a system parameter that must be configured. Hence, for each one of the valid values of D_2 we executed a simulation run of 4000 realizations since there is no other way to calculate the optimal D_2^* except an

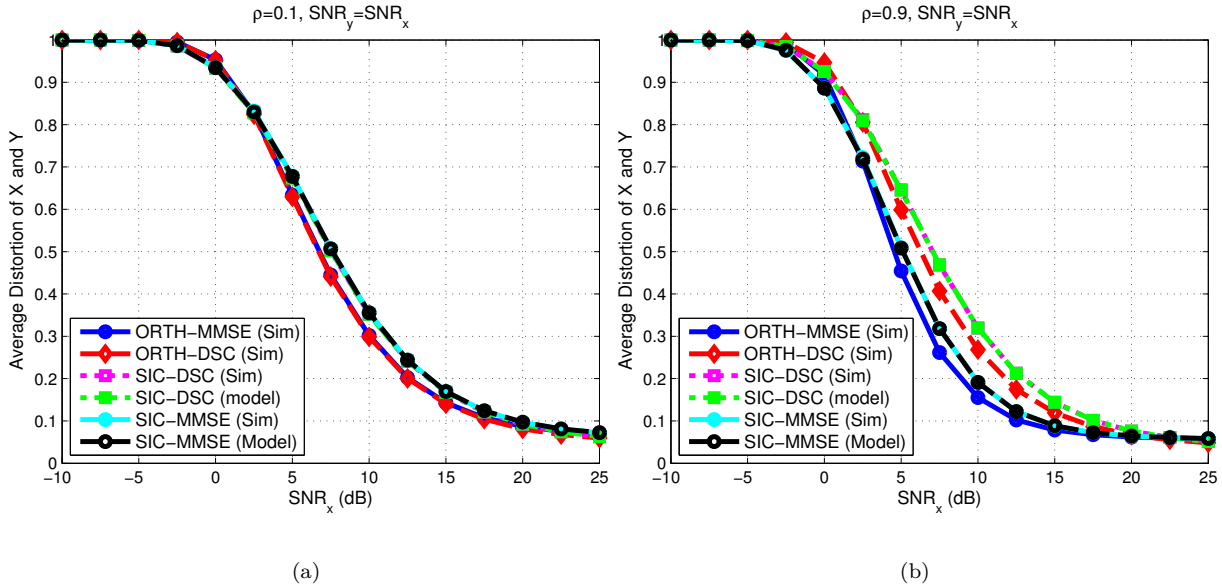


Fig. 4. Simulation and numerical results for the average distortion for symmetric average SNRs ($SNR_x = SNR_y$).

exhaustive search. Thus, we plot the best result from this number of runs (denoted in the figures as DSC (Sim)). Although such an approach is not practical, it provides the optimal value for our setup.

The results to validate the proposed models were obtained with the help of (30) and the similar equations for ED_y^{MMSE} , ED_x^{DSC} , and ED_y^{DSC} . For the MMSE based schemes, the results are denoted in the figures as MMSE (Model) and similarly for the DSC based schemes, the results are denoted as DSC (Model). For the analytical results of the DSC system the optimal D_2^* now was also obtained from the closed form expressions by minimizing $ED_x^{\text{DSC}} + ED_y^{\text{DSC}}$ for the given input parameters presented in the start of the last paragraph.

The MSE distortion of the MMSE-based schemes are illustrated in Fig. 4, where the scenario with symmetric average SNRs ($SNR_x = SNR_y$) has been evaluated. We observe that SIC-MMSE is generally inferior to ORTH-MMSE. For both ORTH-MMSE and SIC-MMSE, we observe that the dominant factor that affects their performance is the average channel SNR. Typically channel symmetry is undesirable for SIC since signals that are received with approximately the same power level, are difficult to be decoded under SIC [9]. For the DSC-based systems and when ρ is high then the impact of a packet losses is more critical for decoding. That is why DSC generally under-performs when compared to MMSE regardless of the use of ORTH or SIC.

For different channel SNR conditions with $SNR_y = 0$ dB in Fig. 5, we notice that SIC-MMSE always

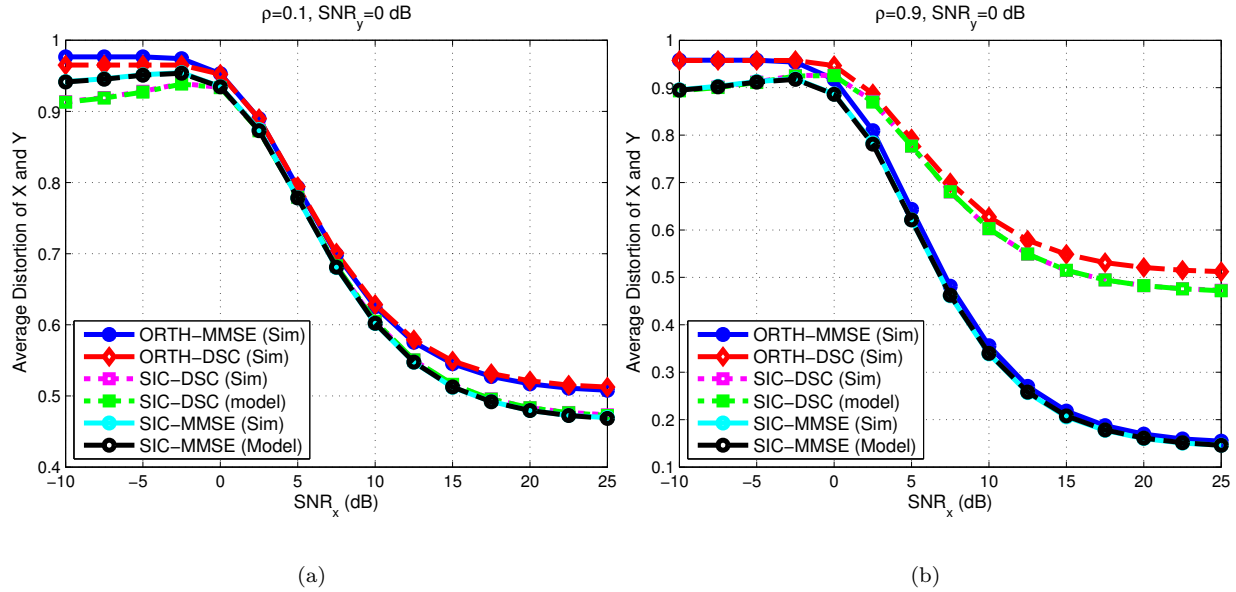


Fig. 5. Comparative simulation and numerical results for the average distortion for different correlation coefficients and $SNR_y=0$ dB.

outperforms ORTH-MMSE regardless of the value of the correlation coefficient. However, the correlation coefficient has an impact on the actual value of the distortion since for $\rho=0.1$ the difference is more important. This behavior is due to SIC that performs very well in asymmetric channel conditions since it can cancel more efficiently the interfering packets [9]. However, the DSC scheme performs even worse than the previous set of results, even when SNR_x is increased. Recall that the decoding of X depends on the successful decoding of the signal Y . In this case, Y can only be communicated at a lower rate, or with higher outage probability, since $SNR_y=0$ dB. Consequently, this result for our particular setup highlights a JSCC trade-off (not exploiting correlation with compression at the sources may help in a lossy network).

For SNR_y equal to 10 dB in Fig. 6 we notice the interesting result that the performance boundaries for the two MMSE schemes have a similar trend but differ in a specific aspect. In particular for $\rho=0.1$ SIC-MMSE is better than ORTH-MMSE for values of SNR_x between 4dB and 17.5dB and for $\rho=0.9$ in the regime 2.5dB and 20dB. Hence, even though the actual value of the average MSE depends on ρ , when we notice the relative performance of SIC-MMSE versus ORTH-MMSE there is also a dependence on the correlation coefficient. A look in our analytical formulas can clarify why the above happens. For a given SNR_y , and as SNR_x is increased, the $P^{(i)}$ values change depending on the average channel conditions. However, in our model these outage probabilities are multiplied with distortion terms that depend on the

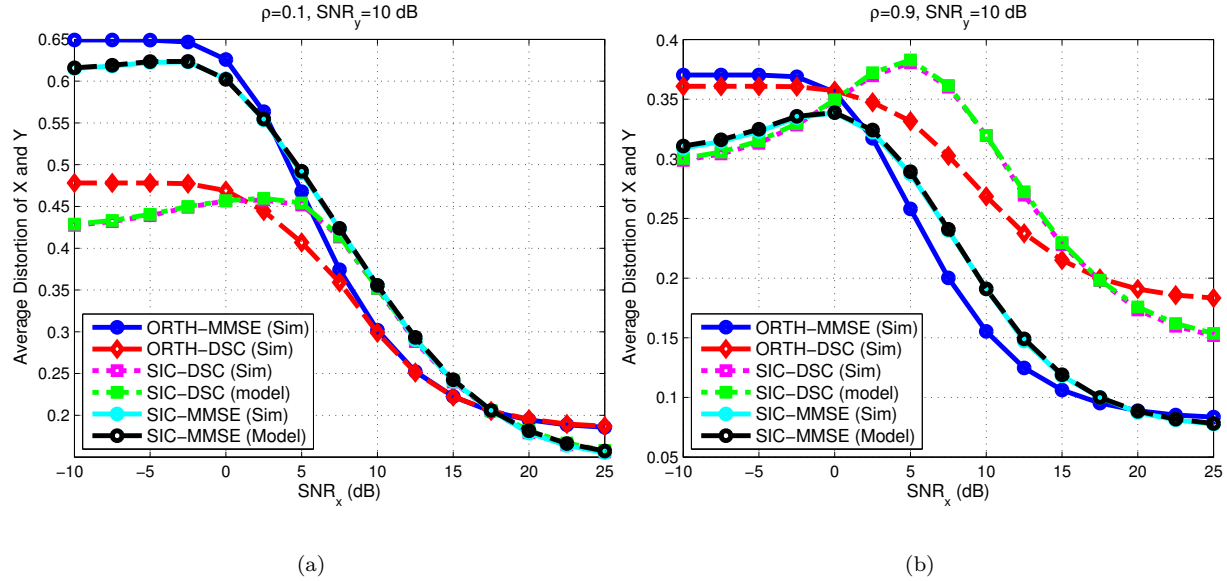


Fig. 6. Comparative simulation and numerical results for the average distortion for different correlation coefficients and $SNR_y=10$ dB.

correlation coefficient. Regarding the performance of DSC-based schemes $\rho=0.1$, and $SNR_y=10$ dB in Fig. 6 we have different performance SNR boundaries for ORTH-DSC and SIC-DSC. The correlation coefficient has a more dominant impact on optimality of the different schemes than the MMSE case. In particular, for $\rho=0.9$, SIC-DSC outperforms ORTH-DSC from around 17.5 dB and beyond, while this is not the case for $\rho=0.1$, where they both converge to the same MSE value. Thus, even when only the average channel is known, the transmission mode and the configuration of the DSC system (expressed through the selection of D_2) should be jointly selected for optimality. There is no effect of shifting performance boundaries like MMSE.

Considering the performance of a particular transmission mode ORTH or SIC for different schemes DSC and MMSE, we observe that ORTH-MMSE has fairly consistent behavior. Similarly ORTH-DSC. For the SIC schemes they are better than ORTH for the MMSE case again when we have asymmetries in the channel SNR. For DSC the situation is similar only in this case there is the dependence of the parameter D_2 .

Finally, the general trend of the results is that there is a very close match between closed-form model and simulation. One of the key reasons is that the outage model accounts for the error propagation in the SIC system. Since this is accurately modeled, the outage expressions correspond to actual average number

of received packets at the receiver for each source. Consequently the average MSE distortion is modeled precisely because it depends on the number of available observations. This is the strength of the proposed model that couples accurately these two aspects.

From these set of results we can also see that even for constant channel conditions and power, the proposed model can be used for selecting the optimal combination of source coding and transmission mode. The full potential of our model for JSCC is investigated next.

B. Numerical Optimization Results

The analytical expressions we developed offer a tool for JSCC optimization by selecting the transmission options, β , the transmit power, and the several remaining system parameters. We present here results for the JSCC formulations presented in Section V.

We first present the numerical optimization results for MSE versus ρ in Fig. 7(a) after solving (31). The only constraint for this optimization is the power budget $\varphi=1$ and all the other parameters are optimized. Note also that the average SNR here is also an optimization parameter for each source that controlled through the allocated power levels φ_x, φ_y . Now for a given power budget φ and data correlation, the optimization in (31) allows us to identify what is the optimal combination of source coding/decoding schemes (MMSE or DSC) and transmission modes (SIC or ORTH), also the optimal source coding rate for each source \bar{R}_x, \bar{R}_y , the optimal power, and optimal modulation scheme. The results in the previous subsection suggested that for a fixed transmit power, or average SNR from the two sources, in certain cases ORTH is better than interference. Now in our fully optimized system, since transmit power and the modulation scheme (R_x, R_y) can be controlled, we notice that interfering transmissions is always the best option. However, the optimality of source coding is something that depends on ρ . In Fig. 7(b) we present results for different variance of the source X. In this case DSC-based schemes perform very poorly for the reasons we explained earlier (dependency between the decoding of X and Y). However, SIC still is a better option.

VII. The Case of Layered Sources

When the two sources are encoded with layered source coding, the analysis becomes slightly more involved due to the dependencies between the layers. Still, our analysis for the outage is applicable.

A. Preliminaries

In this section we discuss the case with source is encoded into multiple layers. Before we introduce our scheme we discuss the layered source coding model. The first layered strategy we consider is the progressive

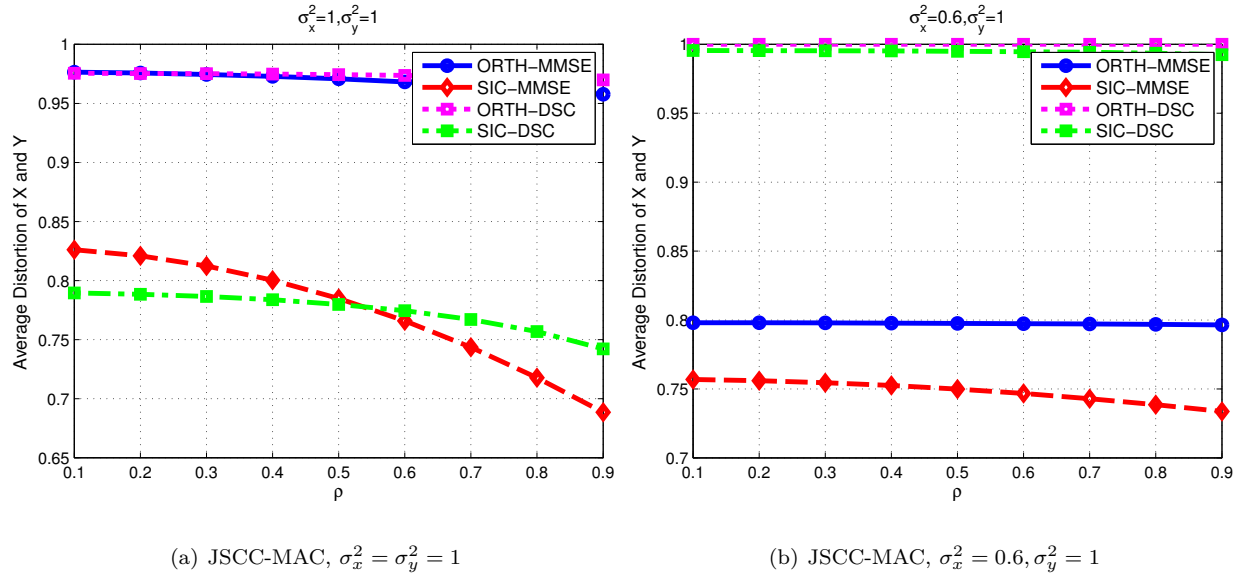


Fig. 7. Results for the optimized average distortion.

transmission (denoted as LS) [24] for two layers in which the base layer is transmitted at a channel rate of R_1 bpcu and a fraction αN of the channel uses (with $0 \leq \alpha \leq 1$) are used. In the second portion, the enhancement layer consisting of the successive refinement bits [25] of the source at a rate of R_2 bpcu is transmitted.

In the case that the base layer (BL) and the enhancement layer (EL) are received, the achieved rate is $\alpha R_1 b + (1 - \alpha) R_2 b$. In the case of an outage event in the enhancement layer, the achieved rate is $\alpha R_1 b$ bits per source sample. Using the successive refinability property, these rates correspond to distortions of $D(\alpha R_1 b + (1 - \alpha) R_2 b)$ and $D(\alpha R_1 b)$, respectively, where $D(R)$ is the distortion rate function of the Gaussian source. In case of an outage of the base layer the EL is also in outage (w.p. P^{BL+EL}), and the achieved distortion is $D(0)$. The expected distortion expression for 2-level LS can be written as [24]:

$$\begin{aligned}
 E[D] &= (1 - P^{EL+BL})D(\alpha R_1 b + (1 - \alpha) R_2 b) \\
 &\quad + (P^{EL+BL} - P^{BL})D(\alpha R_1 b) + P^{BL}D(0)
 \end{aligned} \tag{32}$$

The second strategy we investigate for transmitting the layered source is the broadcast strategy (BS) [26] for two layers. The base layer is transmitted using $\gamma\phi$ power at a channel rate of R_1 bpcu and the enhancement layer using $(1 - \gamma)\phi$ power at a channel rate of R_2 bpcu. Also, the power assignment rule is denoted by γ ($0 \leq \gamma \leq 1$). The receiver attempts to decode the base layer first, as it reads the enchantment layer as

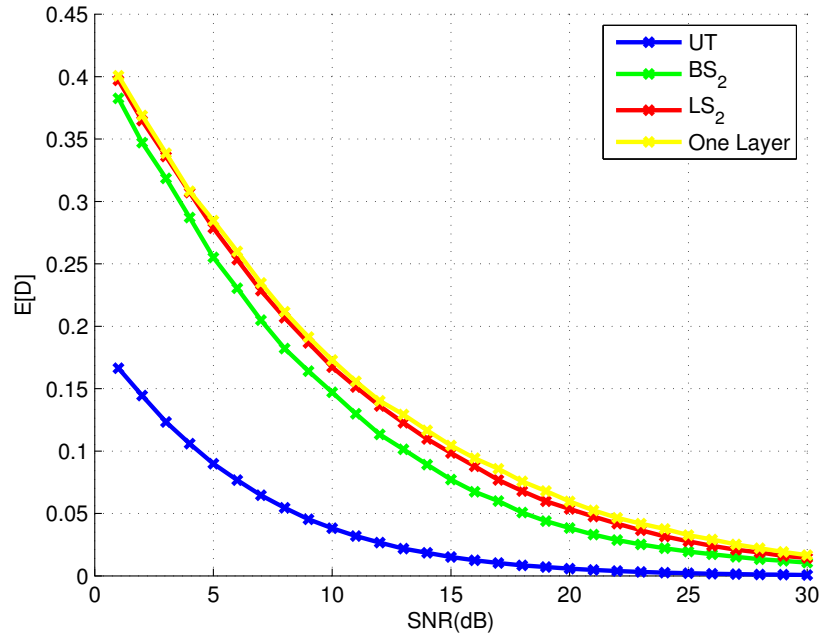


Fig. 8. Expected distortion vs. SNR plots. The topmost curve one layer corresponds to direct transmission without layering.

noise. In the case that the receiver fails to decode the base layer successfully, the achieved rate is equal to zero and the distortion is $D(0)$. If there is a successful decoding of the base layer, it is subtracted from the received signal and the receiver attempts to decode the enhancement layer. In the case of an outage the achieved rate is equal to bR_1 bits per source sample and the distortion is $D(bR_1)$. Otherwise the achieved rate is equal to $bR_1 + bR_2$ bits per source sample and the distortion is $D(bR_1 + bR_2)$. Then, the expected distortion expression for 2-level BS can be written as [24]:

$$\begin{aligned}
 E[D] &= (1 - P^{BL+EL})D(R_1b + R_2b) \\
 &\quad + (P^{BL+EL} - P^{BL})D(R_1b) + P^{BL}D(0)
 \end{aligned} \tag{33}$$

In Fig. 8 we illustrate the performance of these strategies with the case of one layer which is equal to the direct transmission and with the uncoded transmission (UT) which is optimal for an additive white Gaussian channel and a source with squared-error distortion metric with $b = 1$ [27].

Note that the encoder-decoder pair required for LS is simpler than the ones required for BS, because BS requires SNR dependent power allocation among layers, superimposition of codewords and sequential decoding.

B. Proposed Joint Layered Source DSC with Orthogonal Transmission

In this section we propose a novel compression and transmission scheme where Y is compressed separately into multiple layers as we described in the previous subsection, and X is compressed with respect to Y with DSC. First, we consider the scenario with the multiple layer strategy which is used for Y is the progressive transmission (LS) for two layers. We assume that source X transmits at a channel rate of R_x bpcu with corresponding compression rates $\bar{R}_x = bR_x$, and that source Y transmits the base layer at a channel rate of R_{y1} and the enhancement layer, consisting of the successive refinement bits of the source at a rate of R_{y2} bpcu.

We define the related notation first. According to our previous DSC analysis, if the base layer of Y is lost, we denote the target distortion for X by D_2 , otherwise, if the enhancement layer of Y is lost and the base layer is received successfully, we denote the target distortion for X by D_2' . The probability of receiving successfully both the compressed information of X and Y sources is denoted again by $P^{(1)}$, the probability of receiving successfully the compressed information of X and the base layer of Y sources by $P^{(2)}$, the probability of receiving successfully only the compressed information of X source by $P^{(3)}$, the probability of receiving successfully only the compressed information of Y source by $P^{(4)}$, the probability of receiving successfully only the base layer of Y source by $P^{(5)}$, and the probability of failure to receive the compressed information of both X and Y sources by $P^{(6)}$.

Then the distortions for our strategy can be expressed in terms of error/success probabilities as follows:

$$\begin{aligned}
 ED_x &= P^{(1)} D_1(\bar{R}_x, D_2, D_y(\alpha\bar{R}_{y1} + (1-\alpha)\bar{R}_{y2})) \\
 &\quad + P^{(2)} D_1(\bar{R}_x, D_2', D_y(\alpha\bar{R}_{y1})) + P^{(3)} D_2 \\
 &\quad + P^{(4)} D_1(0, D_2, D_y(\alpha\bar{R}_{y1} + (1-\alpha)\bar{R}_{y2})) \\
 &\quad + P^{(5)} D_1(0, D_2', D_y(\alpha\bar{R}_{y1})) + P^{(6)} \sigma_x^2
 \end{aligned} \tag{34}$$

$$\begin{aligned}
 ED_y &= (P^{(1)} + P^{(4)}) D_y(\alpha\bar{R}_{y1} + (1-\alpha)\bar{R}_{y2}) \\
 &\quad + (P^{(2)} + P^{(5)}) D_y(\alpha\bar{R}_{y1}) \\
 &\quad + (P^{(3)} + P^{(6)}) \sigma_y^2
 \end{aligned} \tag{35}$$

Next, we consider a scenario where Y is compressed and transmitted using broadcast strategy (BS) for two layers. We transmit the base layer using $\gamma\phi$ power at a channel rate of R_{y1} bpcu and the enhancement

layer using $(1 - \gamma)\phi$ power at a channel rate of R_{y2} bpcu. The distortions for our strategy can be expressed in terms of error/success probabilities as

$$\begin{aligned}
ED_x &= P^{(1)} D_1(\bar{R}_x, D_2, D_y(\bar{R}_{y1} + \bar{R}_{y2})) \\
&+ P^{(2)} D_1(\bar{R}_x, D_2', D_y(\bar{R}_{y1})) + P^{(3)} D_2 \\
&+ P^{(4)} D_1(0, D_2, D_y(\bar{R}_{y1} + \bar{R}_{y2})) \\
&+ P^{(5)} D_1(0, D_2', D_y(\bar{R}_{y1})) + P^{(6)} \sigma_x^2.
\end{aligned} \tag{36}$$

$$\begin{aligned}
ED_y &= (P^{(1)} + P^{(4)}) D_y(\bar{R}_{y1} + \bar{R}_{y2}) \\
&+ (P^{(2)} + P^{(5)}) D_y(\bar{R}_{y1}) \\
&+ (P^{(3)} + P^{(6)}) \sigma_y^2
\end{aligned} \tag{37}$$

C. Results

Again we assume $\sigma_x^2 = 1$, $\sigma_y^2 = 1$ and $b = 1$. We then consider a symmetric scenario, where the terminals are at an equal distance to the destination. We plot the expected distortion vs. SNR for the direct transmission, LS and BS with 2 layers and UT. The results are numerical by reusing the outage expressions developed before, and by normalizing channel and power allocations in unit. It is clear that BS with two layers outperforms LS for a given bandwidth expansion. In Fig. 9 and 10, we plot the numerical results for (34), (35), (36), and (37) and compare the expected distortions achieved by our proposed combined techniques with the layered sources techniques and DSC for high SNR values. We observe that DSC when combined with BS provides significant reduction to the end-to-end distortion when the correlation of the two sources is high. Moreover, both DSC with LS and DSC with BS outperform DSC in the high SNR regime.

VIII. Discussion and Conclusions

In this paper we developed distortion models for different combinations of compression and transmission strategies that make use of source correlation and interference for Rayleigh fading channels. We showed that the expected distortion of correlated Gaussian sources can be minimized under SIC and orthogonal transmissions for different channel conditions and source coding parameters. Based on the results of the proposed model certain observations can be made.

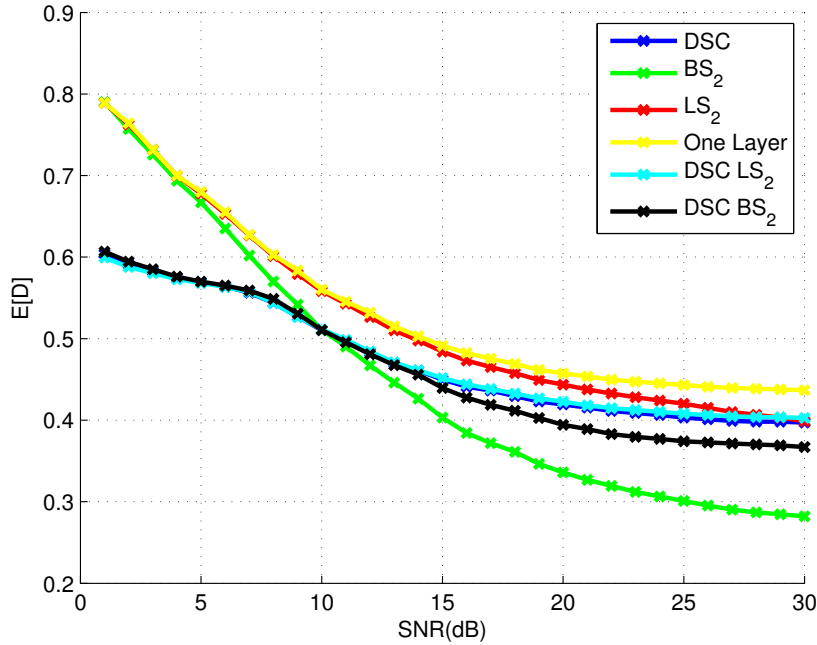


Fig. 9. Expected distortion vs. SNR plots for $\rho = 0.2$.

DSC is sensitive to the availability of side information. When DSC is employed at the application layer it is not always the optimal source coding scheme when the links are asymmetric. Independent compression can be the optimal choice regardless of the transmission scheme (ORTH or SIC).

Furthermore, the well known result that SIC performs best when there are significant differences in the power of the interfering signals does not hold when the performance metric is the signal distortion of correlated data regardless of the source coding scheme. When SIC is optimal for specific channel conditions, we observed that significantly more packets are decoded over ORTH. In this case more observations are available for joint MMSE estimation or DSC decoding. However, this is not enough for calculating (and eventually optimizing) the distortion of the systems since which specific packets are decoded (from which source) is what matters. Thus, the threshold of optimality between ORTH and SIC that we observed in certain simulations, is not simply the threshold that the receiver decodes more data packets (samples) from each source as it would be in a typical multi-user communication scheme based on SIC. In the case of lossy communication of correlated sources this optimality threshold "shifts" in the SNR regime depending on the compression rate at the source \bar{R}_x , the correlation coefficient ρ , and of course the compression scheme.

We also proposed compression and transmission strategies that make use of source correlation with DSC,

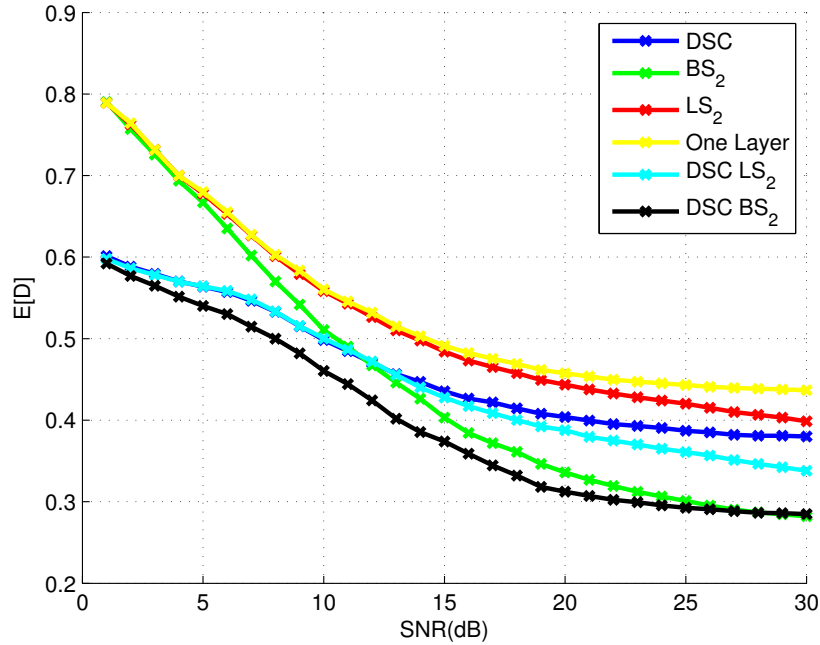


Fig. 10. Expected distortion vs. SNR plots for $\rho = 0.8$.

and layering. We demonstrated that by combining two commonly used techniques (DSC and layered sources), the expected distortion of correlated Gaussian sources can be minimized significantly for high correlation coefficients and low.

There are several potential avenues of future work. One is the design of a system that adapts in real-time and selects the optimal compression scheme (DSC, layered, single layer) and SIC or not depending on the operating regime. Second, is the consideration of multiple data sources in a complete WSN setup with real video.

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